

Digital Signal Processing

Unit -5---IIR FILTER DESIGN

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Overview

- Filters
- Common Filter Types
- Analog vs. Digital Filters
- Difference Equation
- Difference Equation Diagram Elements
- Nonrecursive Difference Equation Diagrams
- Recursive Difference Equation Diagrams
- Impulse Response
- Step Response

Filters

- Filters change a signal's characteristics by selectively removing some of its frequency elements.
- A low pass filter, for example, removes the high frequency components of a signal, but passes low frequency components.
- It does this because the filter's gain, the amplification factor it applies to an input, varies with frequency.

Filters

- For a **low pass filter**, the gains are highest at low frequency and much lower at high frequency.
- A **high pass filter** has exactly the opposite shape: its gains are highest at high frequency and lowest at low frequency.
- The **pass band** of a filter determines the range of frequencies that are passed.
- The **stop band** of the filter determines the range of frequencies that are strongly attenuated.

Filters

Cutoff Frequency

- A filter is considered to pass signals at frequencies where the filter's gain exceeds 0.707 of its maximum gain.
- The frequency or frequencies where the gain equals 0.707 of the maximum gain are called the cut-off frequencies of the filter.
- Since $20\log 0.707 = -3 \text{ dB}$, they are also called -3 dB frequencies.

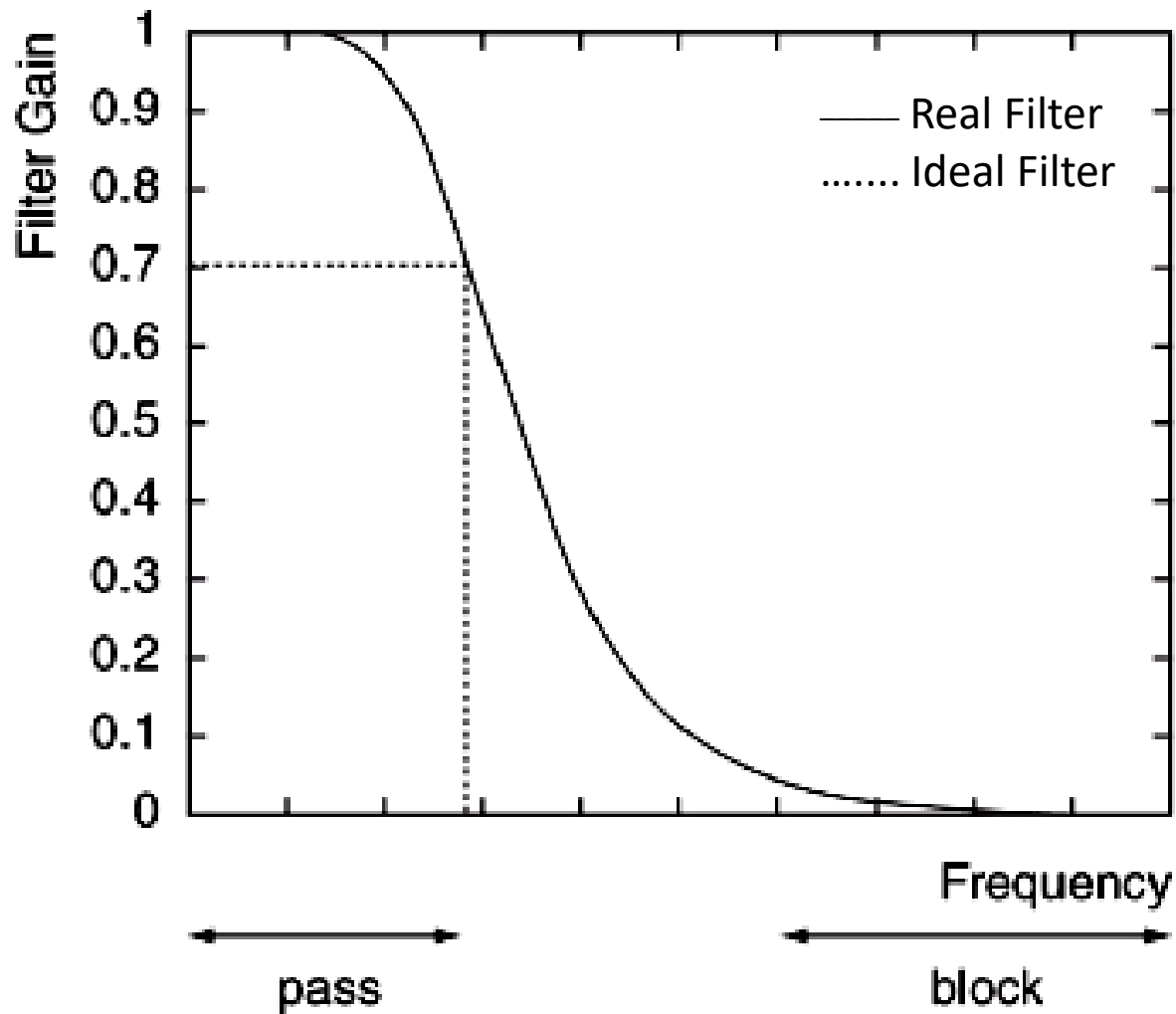
Filters

Bandwidth

- The bandwidth of a low pass filter is the range of frequencies from 0 to the -3 dB frequency.
- For a high pass filter, the bandwidth is the range of frequencies from the -3 dB frequency to half the sampling frequency.
- For band pass filters, the bandwidth is the distance in Hz between the cut-off frequencies.

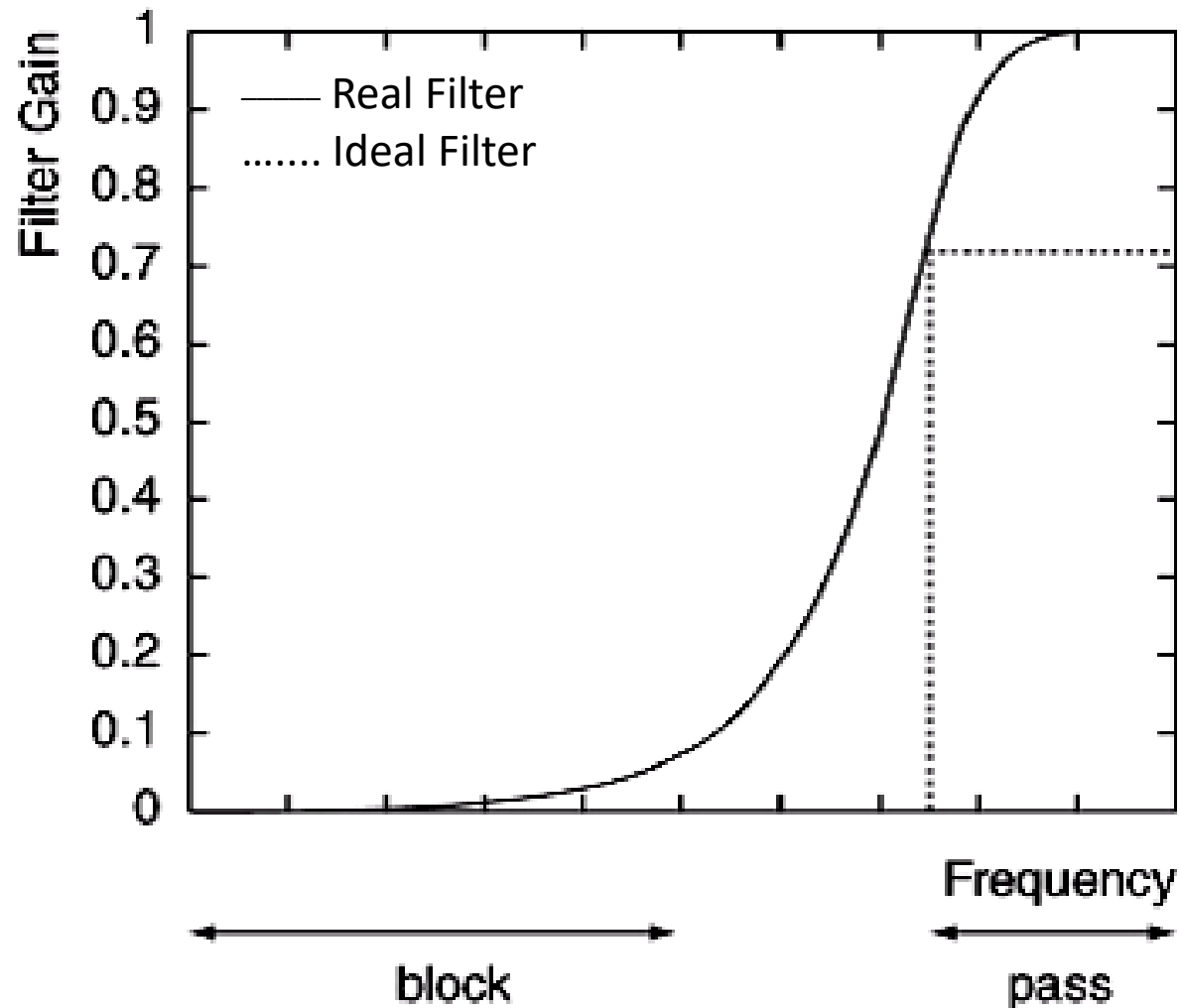
Common Filter Types

Low Pass Filter



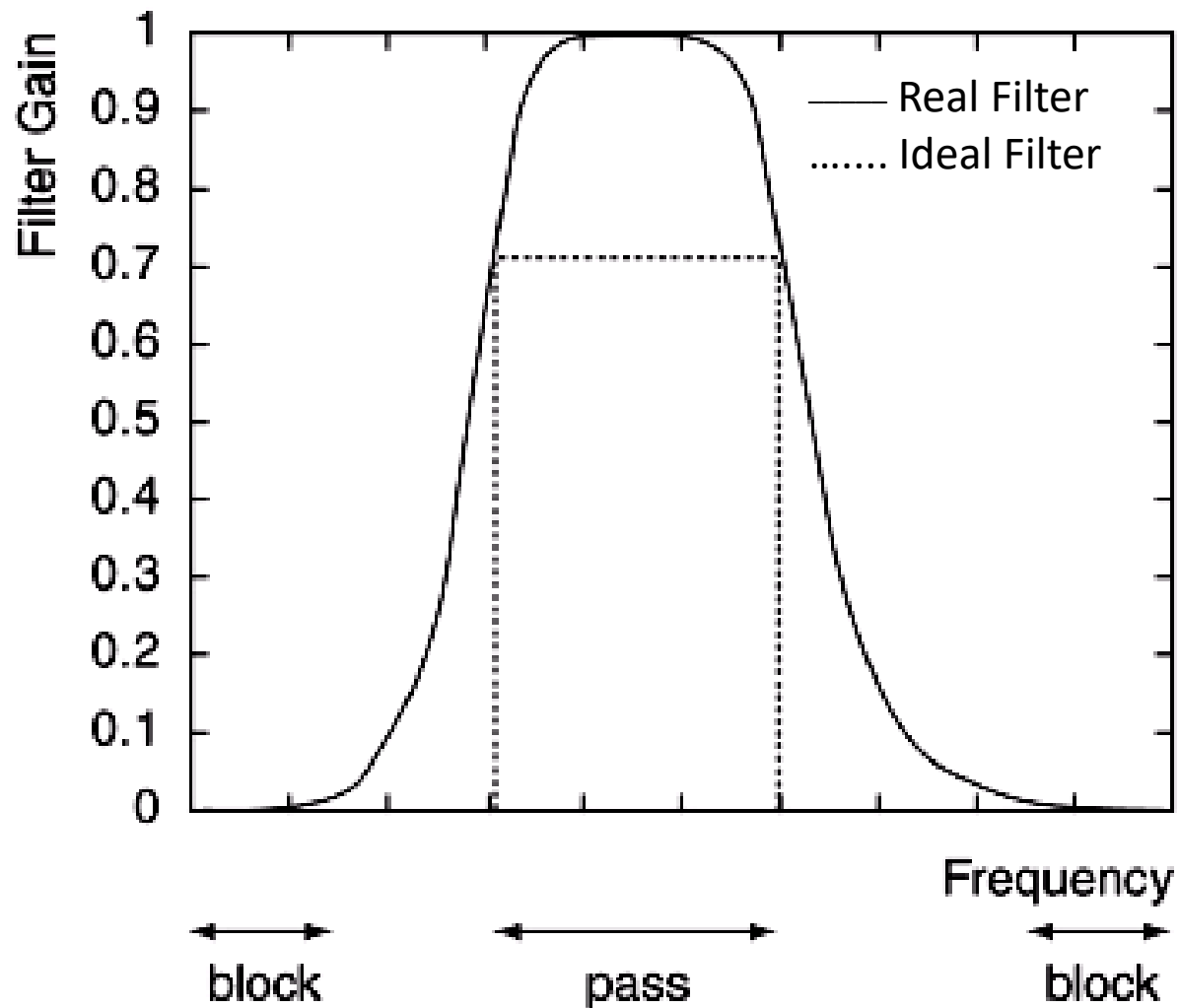
Common Filter Types

High Pass Filter



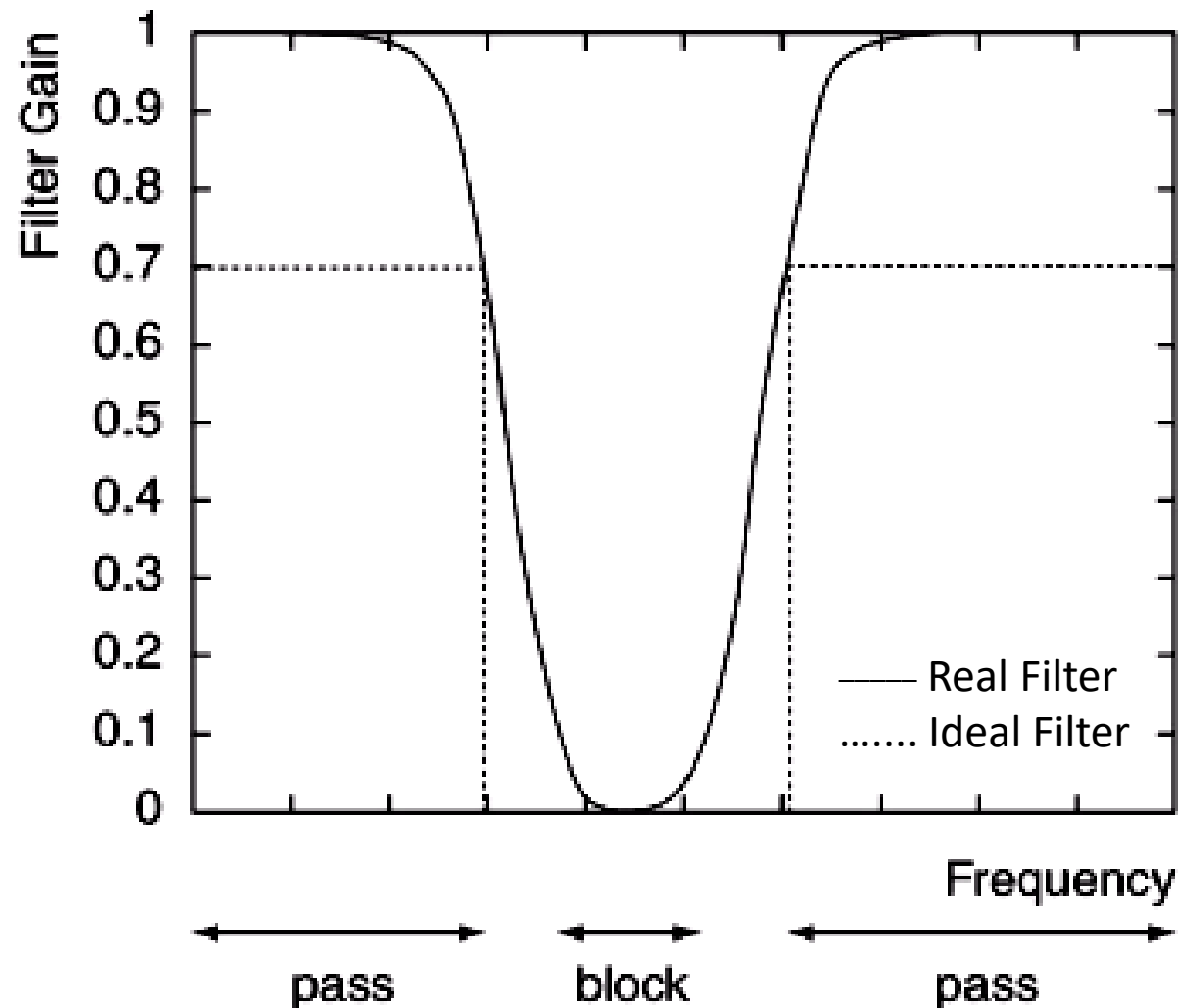
Common Filter Types

Band Pass Filter



Common Filter Types

Band Stop or Notch Filter



Filters

Roll-Off

- Another important feature of a filter is its roll-off.
- This characteristic determines how quickly the gain drops outside the pass band.
- The higher the order of a digital filter, defined by the number of coefficients needed to specify it, the steeper the slope, and the higher the quality of the filter is said to be.

Filters

Gain

- A filter's gain at a certain frequency determines the amplification factor that the filter applies to an input at this frequency.
- A gain may have any value.
- In the pass band region, filter's gain is high.
- In the stop band region, filter's gain is low.

Filters

Gain

- The cutoff frequency of the filter occurs when the gain is $\frac{1}{\sqrt{2}} \sim 0.707$ or 70.7%
- A gain in dB is calculated as $Gain(dB) = 20\log(Gain)$

Filters

Bandwidth

Example-1: Find the bandwidth of the band pass filter.

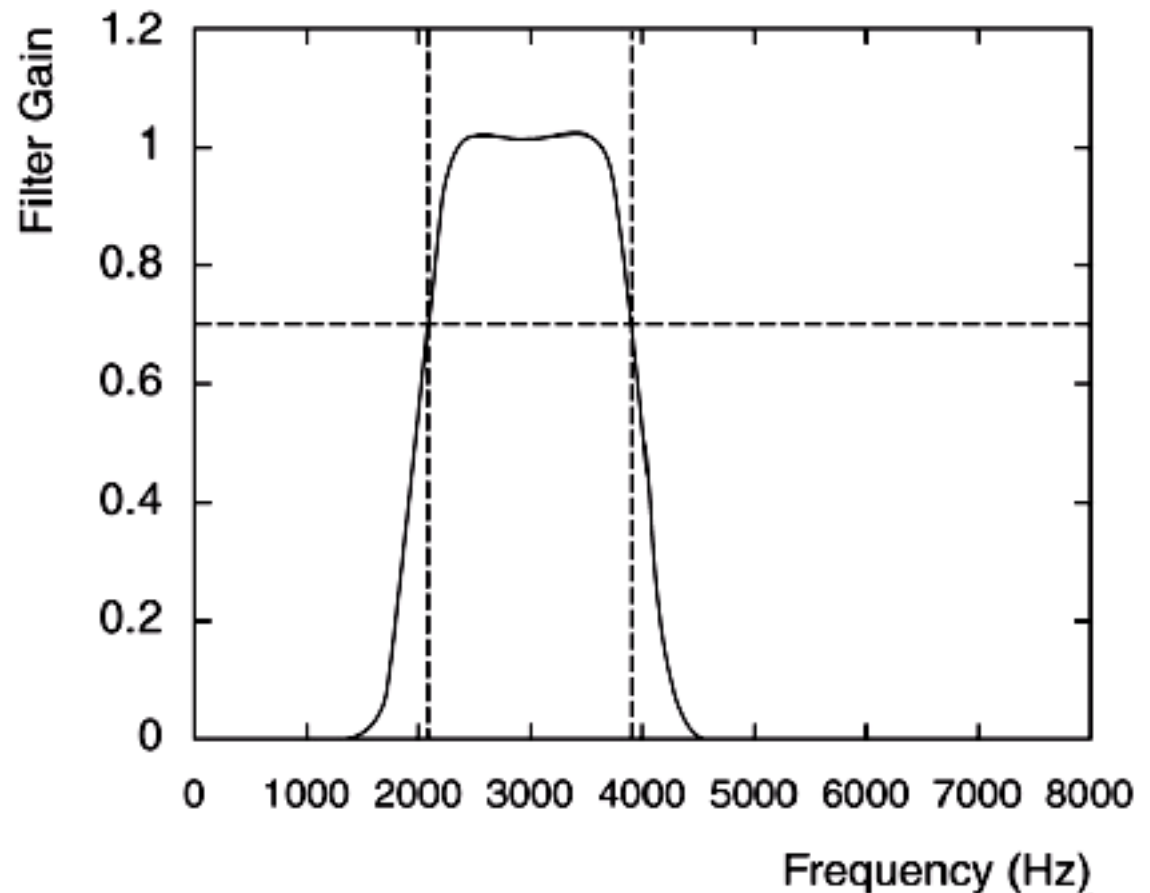
The edges of the pass Band occur where the Gain equals 0.707.

The bandwidth of the Filter is

$$BW = F_H - F_L$$

$$BW = 4000 - 2000$$

$$BW = 2000 \text{ Hz}$$



Analog vs. Digital Filters

- Filters may be implemented in either analog or digital form.
- Analog filters are defined in hardware, while digital filters are defined in software.
- In general, digital filters are much less susceptible to noise and component variation than are analog filters.
- Furthermore, digital filter re-design is simply a matter of editing a list of coefficients, while analog re-design requires building an entirely new circuit.

Difference Equation

- A **difference equation** is one way to specify a filter or system.
- It may be presented in equation or diagram form.
- The general form of a difference equation is

$$a_0y[n] + a_1y[n-1] + a_2y[n-2] + \dots + a_Ny[n-N] \\ = b_0x[n] + b_1x[n-1] + b_2x[n-2] + \dots + b_Mx[n-M]$$

- The a_k and b_k weightings are called filter coefficients.
- Generally difference equation has $N+1$ a_k coefficients and $M+1$ b_k coefficients.
- N is the past output required, also referred as a filter order.
- M is the number of past input required.
- The equation defines how each new output $y[n]$ is obtained.
- In general, a_0 is assumed to be one.

Difference Equation

- The general form of a **difference equation** is

$$\begin{aligned} a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] + \dots + a_N y[n-N] \\ = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \dots + b_M x[n-M] \end{aligned}$$

The compact form of the recursive difference equation is

$$y[n] = -\sum_{k=1}^N a_k y[n-k] + \sum_{k=0}^M b_k x[n-k]$$

- In the above equation, **both inputs and outputs are needed to compute a new output**, so the difference equation is said to be **recursive**.

Difference Equation

- When only inputs are needed to compute a new output, the difference equation is said to be **non-recursive**, that is,

$$y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2] + \dots + b_Mx[n-M]$$

The compact form of the non-recursive difference equation is

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

Difference Equation

Example-2: A filter has a difference equation $y[n] = 0.5y[n-1] + x[n]$

- a. Is this a recursive or non-recursive difference equation?**
- b. Identify all a_k and b_k coefficients.**
- c. If the input $x[n]$ is $u[n]$, find 5 samples of the output $y[n]$.**

Solution

a. The equation is recursive as it depends on a past output $y[n-1]$

b. Since $y[n] - 0.5y[n-1] = x[n]$, therefore $a_0 = 1$, $a_1 = -0.5$, $b_0 = 1$

c. $y[n] = 0.5y[n-1] + x[n]$

When **$n = 0$** , $y[0] = 0.5y[0-1] + x[0] = 0.5(0) + 1 = 1$

When **$n = 1$** , $y[1] = 0.5y[1-1] + x[1] = 0.5(1) + 1 = 1.5$

When **$n = 2$** , $y[2] = 0.5y[2-1] + x[2] = 0.5(1.5) + 1 = 1.75$

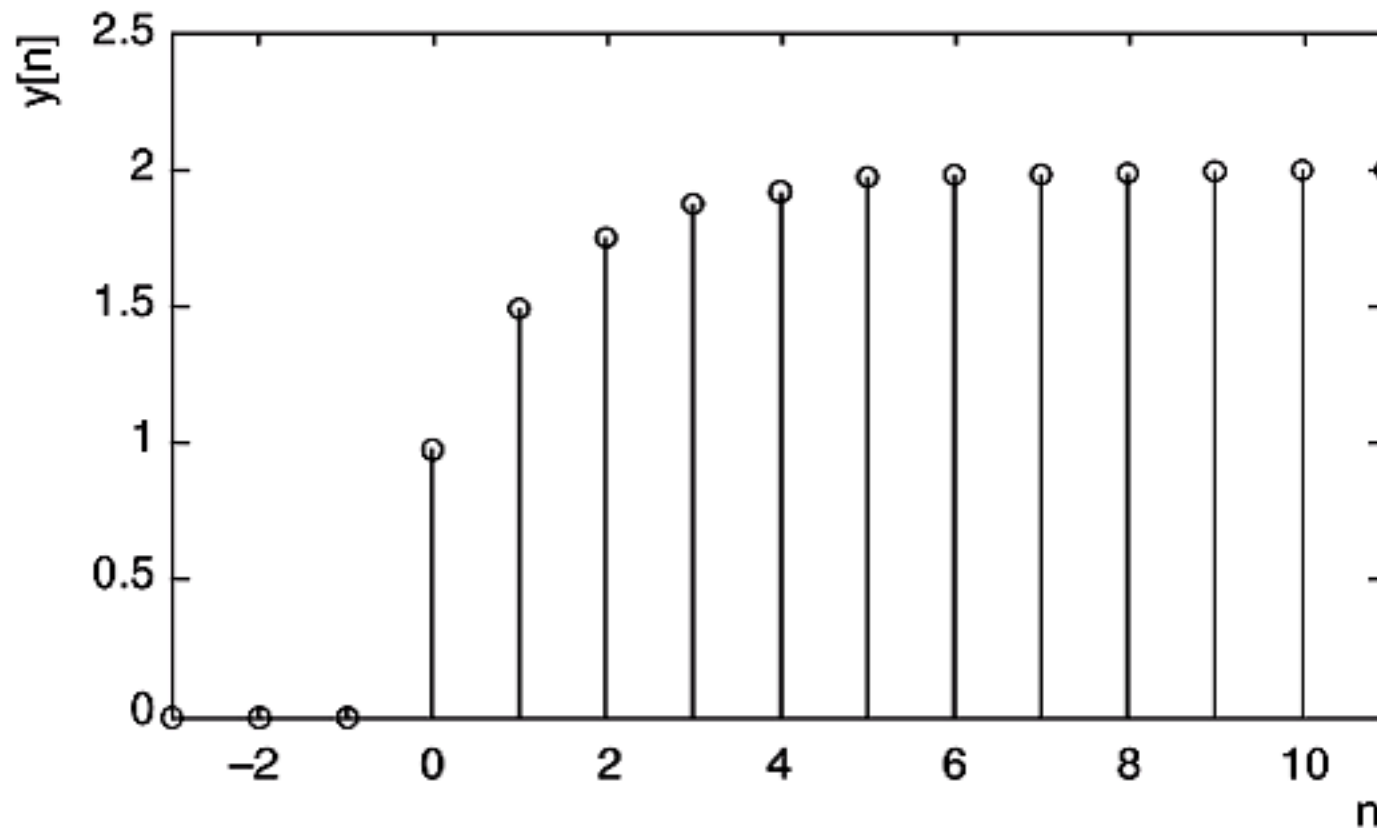
When **$n = 3$** , $y[3] = 0.5y[3-1] + x[3] = 0.5(1.75) + 1 = 1.875$

When **$n = 4$** , $y[4] = 0.5y[4-1] + x[4] = 0.5(1.875) + 1 = 1.9375$

$$Y[n] = \{1, 1.5, 1.75, 1.875, 1.9375\}$$

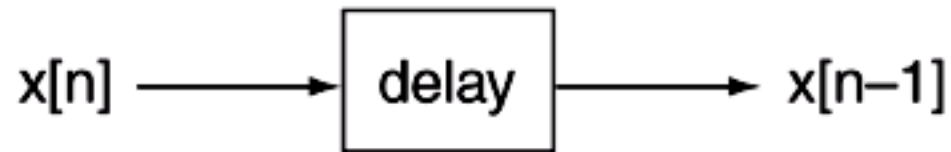
Difference Equation

$$Y[n] = \{1, 1.5, 1.75, 1.875, 1.9375\}$$



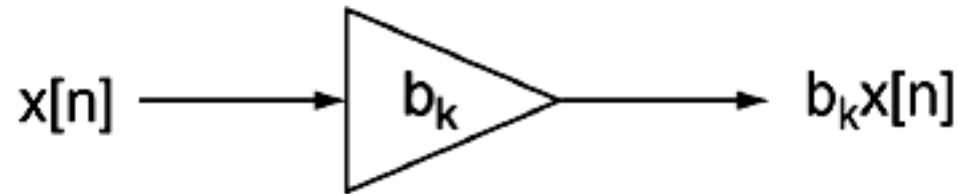
Difference Equation Diagram Elements

Delay Element



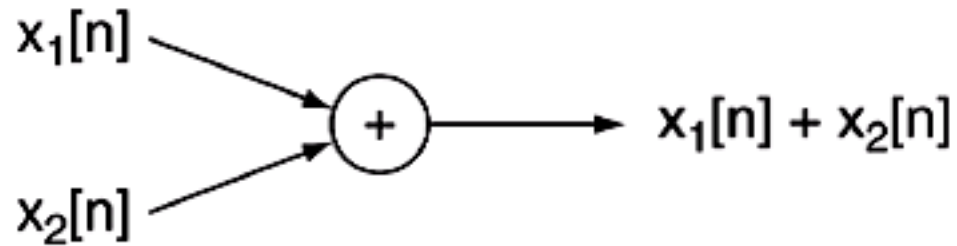
Difference Equation Diagram Elements

Coefficient Multiplier Element



Difference Equation Diagram Elements

Summer Element



Overview

- Realization of Digital Systems
- Direct-form I Realization
- Direct-form II Realization
- Cascade or Series Realization
- Parallel Realization

Realization of Digital Systems

- A **realization** is a hardware or software configuration that implements the system.
- Digital filters described by the transfer function $H(z)$ may be generally realized into the following forms:
 - Direct-form I
 - Direct-form II
 - Cascade or Series
 - Parallel
- There are other forms of realizations such as lattice, however in this course we will only study above mentioned realizations.

Realization of Digital Systems

- The transfer function $H(z)$ of the digital filter (system) transfer is given by:

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}} \quad (1)$$

- Let $x(n)$ and $y(n)$ be the digital filter (system) input and output, respectively. We can express the relationship in z-transform domain as:

$$Y(z) = H(z)X(z) \quad (2)$$

Realization of Digital Systems

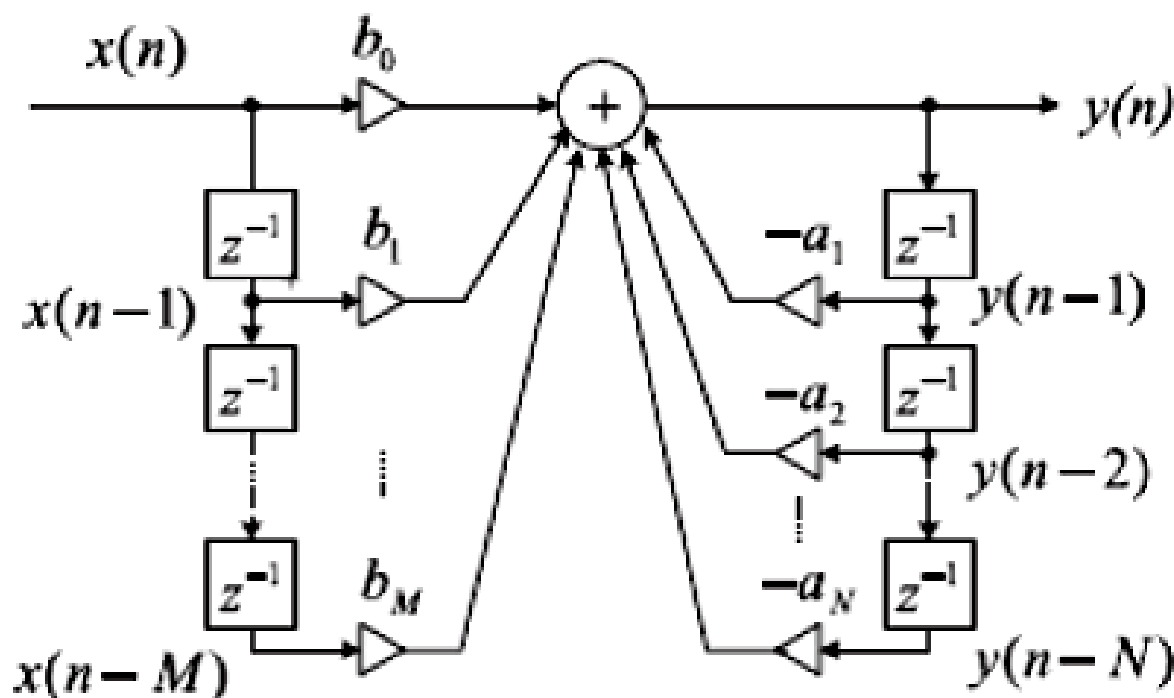
- By substituting $H(z)$ from equation (1) in (2) yields
- Taking the inverse of the z -transform of Equation (3) we get the difference recursive difference equation

$$y(n) = b_0x(n) + b_1x(n-1) + \cdots + b_Mx(n-M) - a_1y(n-1) - a_2y(n-2) - \cdots - a_Ny(n-N) \quad (4)$$

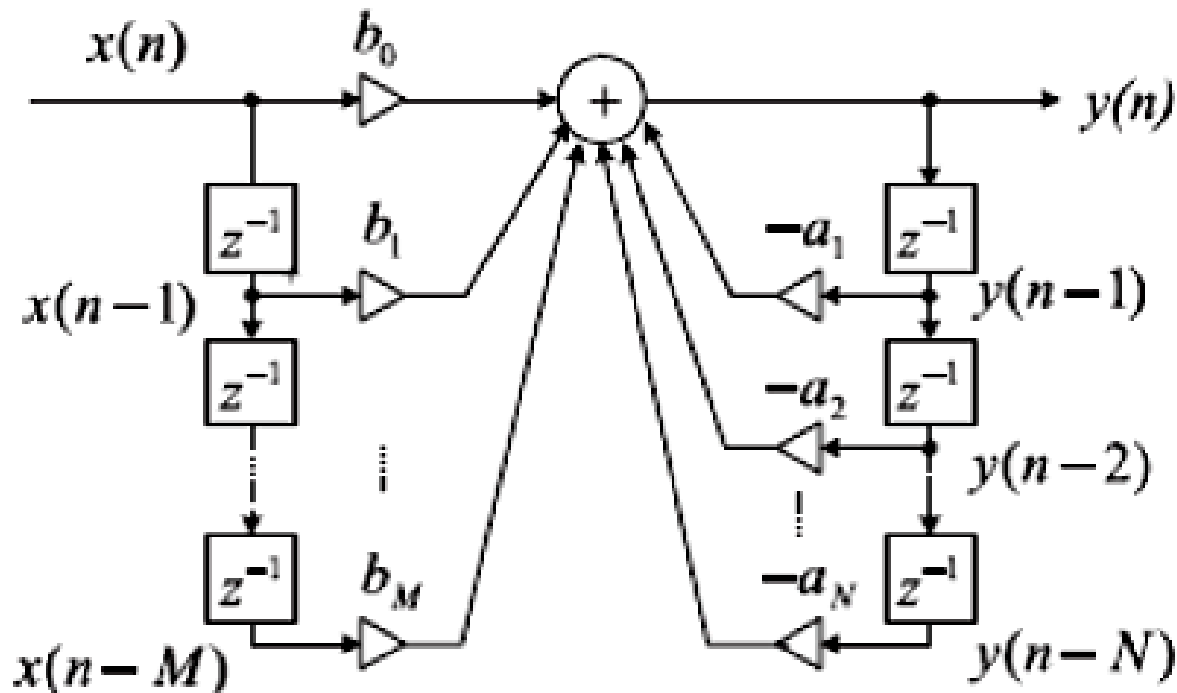
Direct-Form I Realization

$$y(n) = b_0x(n) + b_1x(n-1) + \dots + b_Mx(n-M) - a_1y(n-1) - a_2y(n-2) - \dots - a_Ny(n-N) \quad (4)$$

- The recursive difference equation can be implemented by the **direct-form I realization** as shown in the figure.



Direct-Form I Realization



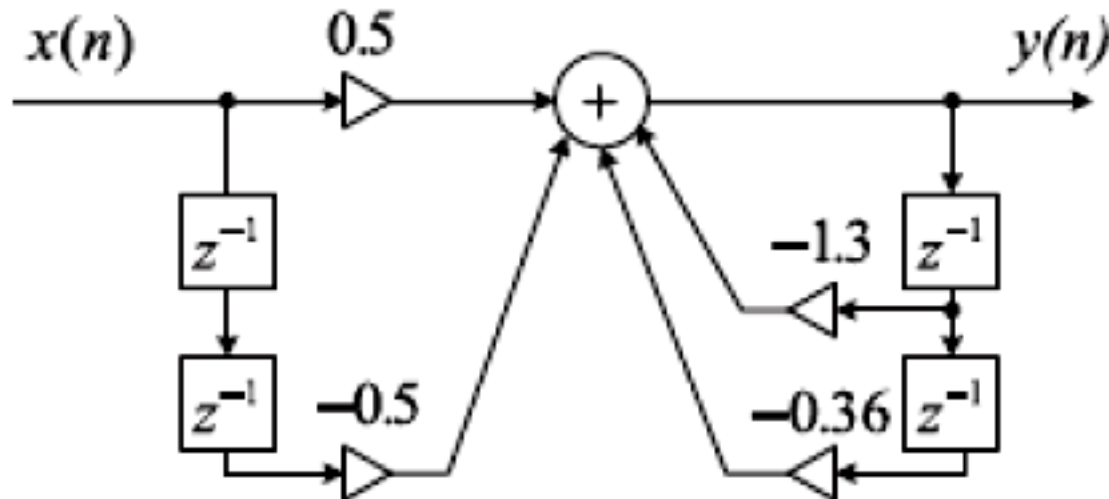
- Calculation for $y[n]$ using this form require $M+1$ input states, N output states, $M+N+1$ coefficient multiplications, and $M+N$ additions.
- When more than two or three delays are needed, this particular implementation is very sensitive to the **finite word length effects**.

Direct-Form I Realization

Example-1: Draw a Direct-Form I realization of a second-order transfer function $H(z)$ given by

$$H(z) = \frac{0.5(1 - z^{-2})}{1 + 1.3z^{-1} + 0.36z^{-2}}$$

Solution



Direct-Form II Realization

- We consider digital filter (system) defined with transfer function, $H(z)$, given by

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}} \quad (5)$$

- Considering $N=M$, we can express equation (5) as

$$\begin{aligned} Y(z) &= H(z)X(z) = \frac{B(z)}{A(z)}X(z) = B(z) \left(\frac{X(z)}{A(z)} \right) \\ &= (b_0 + b_1 z^{-1} + \dots + b_M z^{-M}) \underbrace{\left(\frac{X(z)}{1 + a_1 z^{-1} + \dots + a_M z^{-M}} \right)}_{W(z)} \quad (6) \end{aligned}$$

Direct-Form II Realization

- Also, if we define a new z-transform function as

$$W(z) = \frac{X(z)}{1 + a_1 z^{-1} + \dots + a_M z^{-M}} \quad (7)$$

- We have

$$Y(z) = (b_0 + b_1 z^{-1} + \dots + b_M z^{-M})W(z) \quad (8)$$

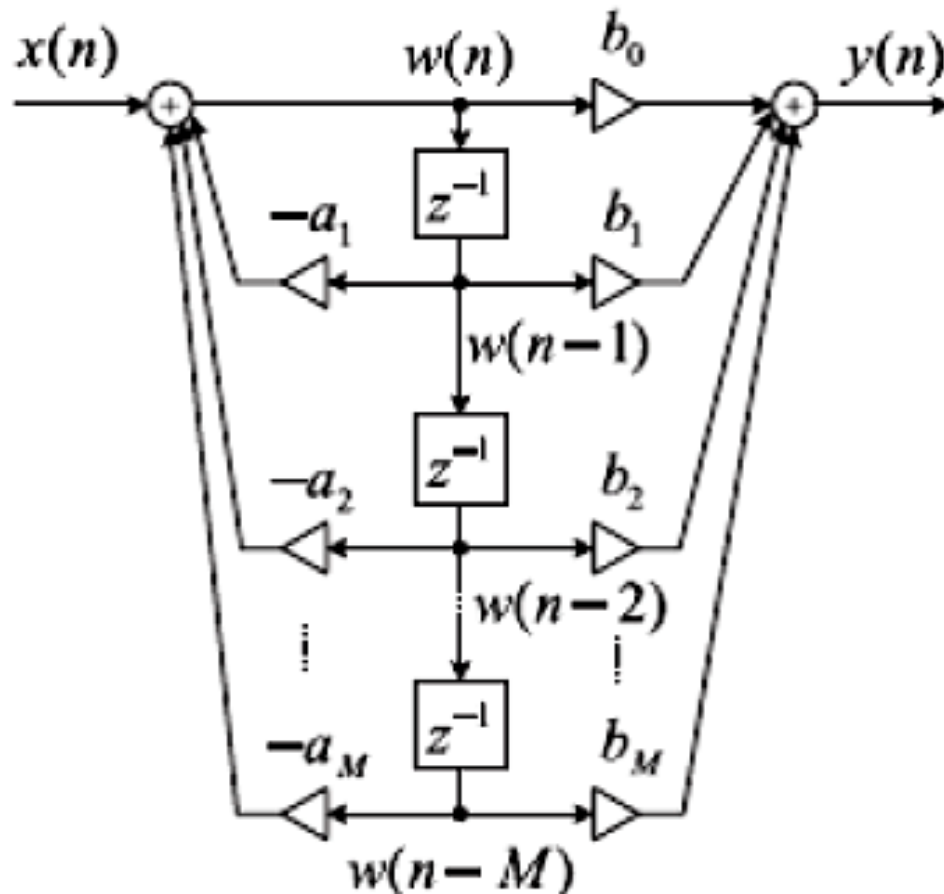
- The corresponding difference equations for Equations (7) and (8), respectively, become

$$w(n) = x(n) - a_1 w(n-1) - a_2 w(n-2) - \dots - a_M w(n-M) \quad (9)$$

$$y(n) = b_0 w(n) + b_1 w(n-1) + \dots + b_M w(n-M) \quad (10)$$

Direct-Form II Realization

- Realization of Equations (9) and (10) produces **direct-form II realization**, (also called **canonical form**) which is demonstrated in following figure.



Direct-Form II Realization

- Much less storage is needed if a direct form II realization is used.
- This realization requires the use of an intermediate signal $w(n)$ that records salient information about the history of the filter in place of past inputs and past outputs.
- Instead of N past outputs and M past inputs, only $\max(N, M)$ samples of the intermediate signal need to be remembered to produce the new filter outputs.
- Calculation for $y(n)$ using this implementation require $\max(N, M)+1$ intermediate states, $M+N+1$ coefficient multiplications, and $M+N$ additions.

Direct-Form II Realization

Example-2: Draw a Direct-Form II realization of a second-order transfer function given by

$$H(z) = \frac{0.5(1 - z^{-2})}{1 + 1.3z^{-1} + 0.36z^{-2}}$$

Solution

The difference equations for the direct-form II realization are expressed as

$$w(n) = x(n) - 1.3w(n-1) - 0.36w(n-2) \quad (11)$$

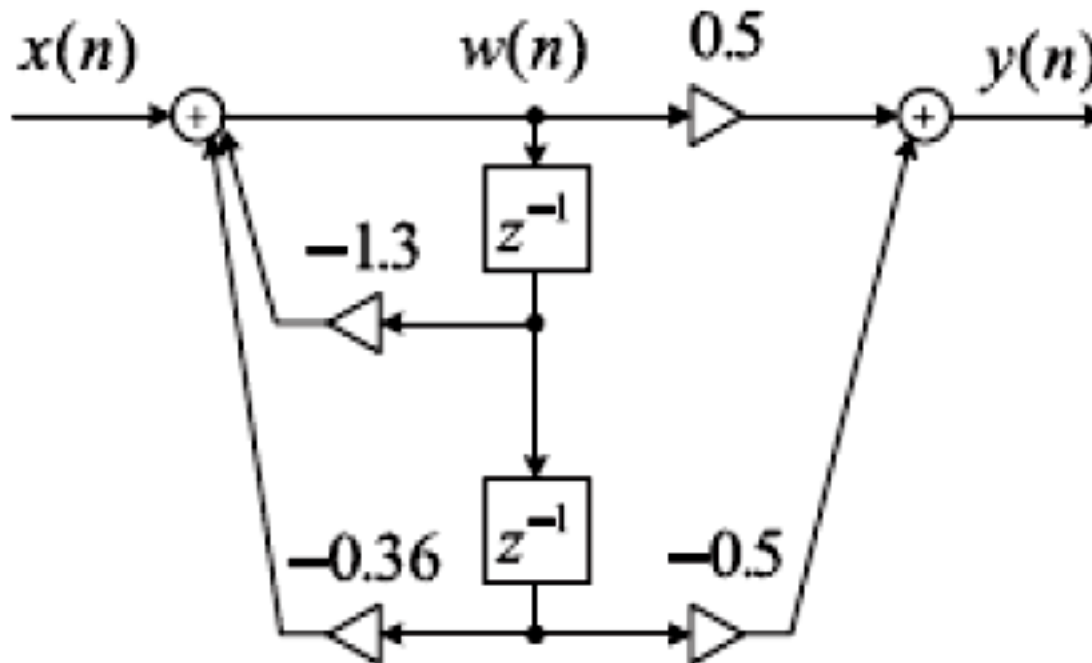
$$y(n) = 0.5w(n) - 0.5w(n-2) \quad (12)$$

Direct-Form II Realization

$$w(n) = x(n) - 1.3w(n-1] - 0.36w(n-2) \quad (11)$$

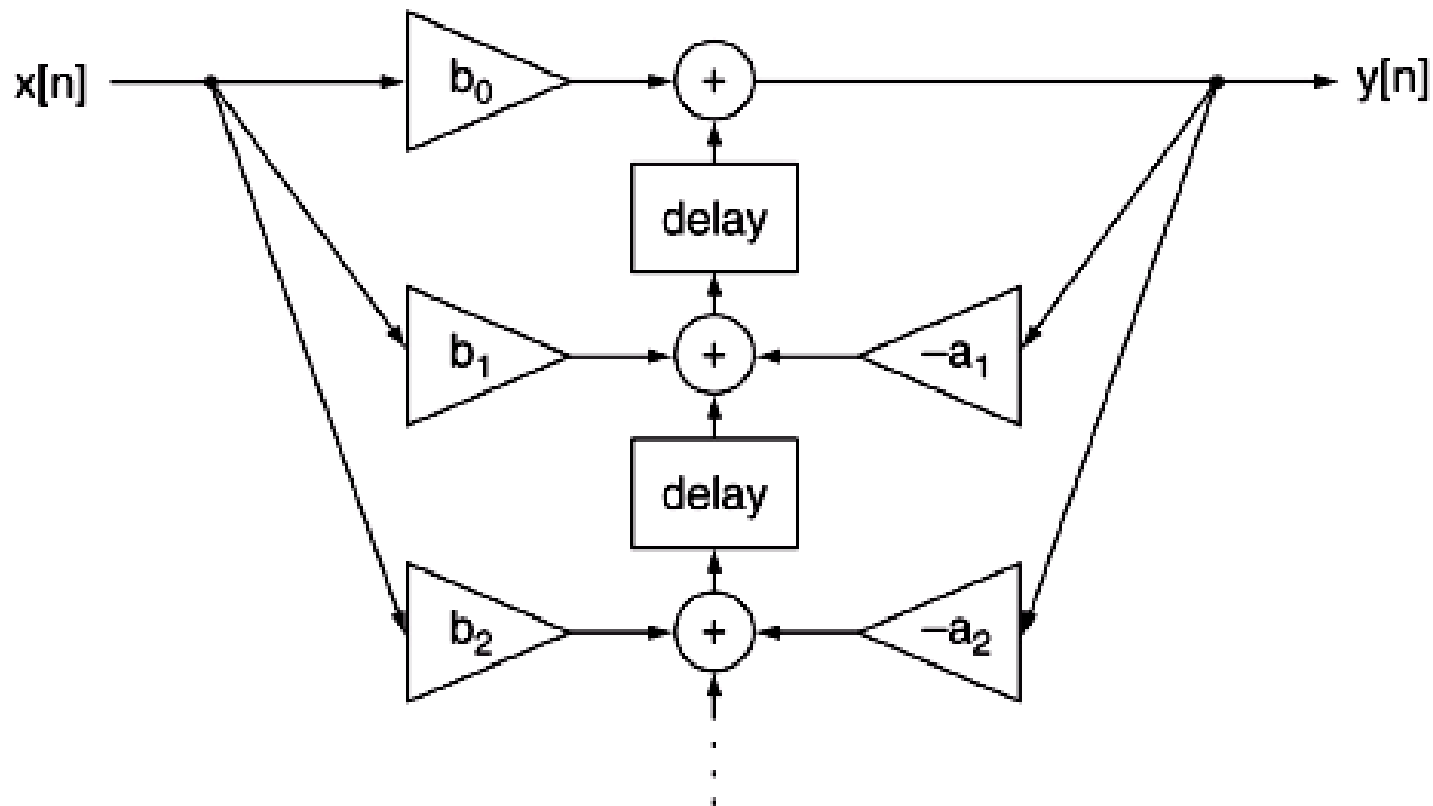
$$y(n] = 0.5w(n) - 0.5w(n-2) \quad (12)$$

The realization is shown in following figure.



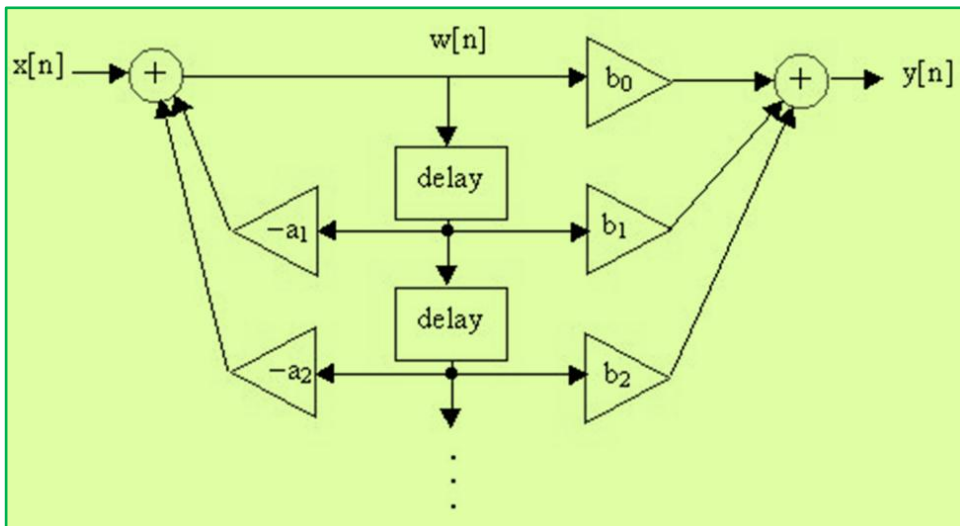
Transpose of the Direct-Form II Realization

- ***Transpose of the direct form II realization*** is another popular implementation model, obtained by reversing the flows of information of the direct form II realization.

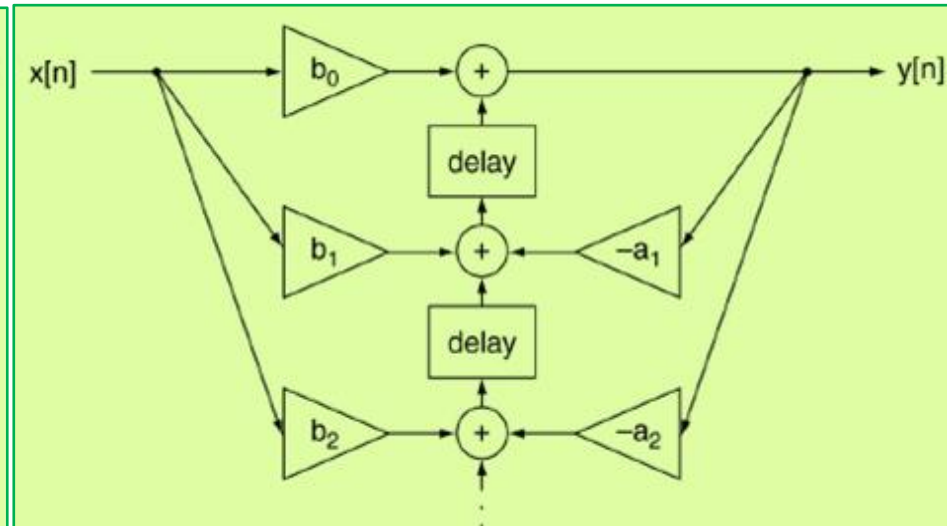


Transpose of the Direct-Form II Realization

- To minimize the finite word length effect, second order blocks of the direct form II realization or its transpose are frequently connected together in cascade or parallel combinations to produce higher order filter implementation.



Direct Form II Realization



Transpose of Direct Form II Realization

Cascade (Series) Realization

- An alternate way to filter realization is to cascade the factorized $H(z)$ in the following form:

$$H(z) = H_1(z) \cdot H_2(z) \cdots H_k(z) \quad (13)$$

where $H_k(z)$ is chosen to be the first- or second-order transfer function (section), which is defined by

$$H_k(z) = \frac{b_{k0} + b_{k1}z^{-1}}{1 + a_{k1}z^{-1}} \quad (14)$$

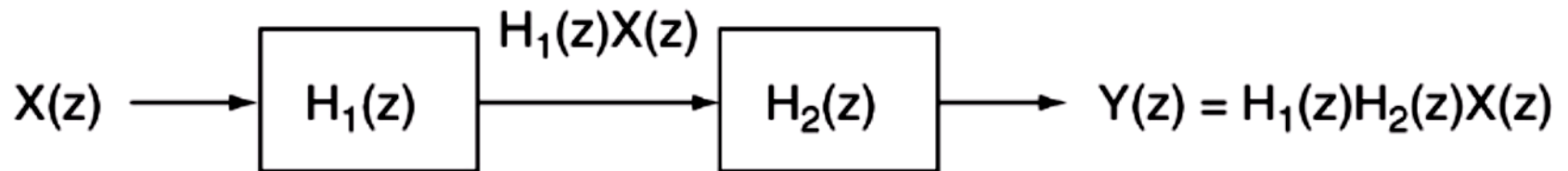
or

$$H_k(z) = \frac{b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}} \quad (15)$$

respectively.

Cascade (Series) Realization

- When two or more systems are combined in cascade (in series), the transfer functions of the systems can be used to determine a transfer function for the overall system.
- E.g. The transfer function of two systems $H_1(z)$ and $H_2(z)$ cascaded together is $H(z) = H_1(z)H_2(z)$
- The block diagram of the cascade, or series, realization is depicted in following figure.



(a) Cascade Combination of Filters

Cascade (Series) Realization

Example-3: Draw a Cascade realization of a second-order transfer function $H(z)$ given by

$$H(z) = \frac{0.5(1 - z^{-2})}{1 + 1.3z^{-1} + 0.36z^{-2}}$$

Solution

To achieve the cascade (series) form realization, we factor $H(z)$ into two first-order sections to yield

$$\begin{aligned} H(z) &= \frac{0.5(1 - z^{-2})}{1 + 1.3z^{-1} + 0.36z^{-2}} \\ &= \left(\frac{0.5 - 0.5z^{-1}}{1 + 0.4z^{-1}} \right) \left(\frac{1 + z^{-1}}{1 + 0.9z^{-1}} \right) \end{aligned}$$

Cascade (Series) Realization

$$= \left(\frac{0.5 - 0.5z^{-1}}{1 + 0.4z^{-1}} \right) \left(\frac{1 + z^{-1}}{1 + 0.9z^{-1}} \right)$$

where $H_1(z)$ and $H_2(z)$ are chosen to be

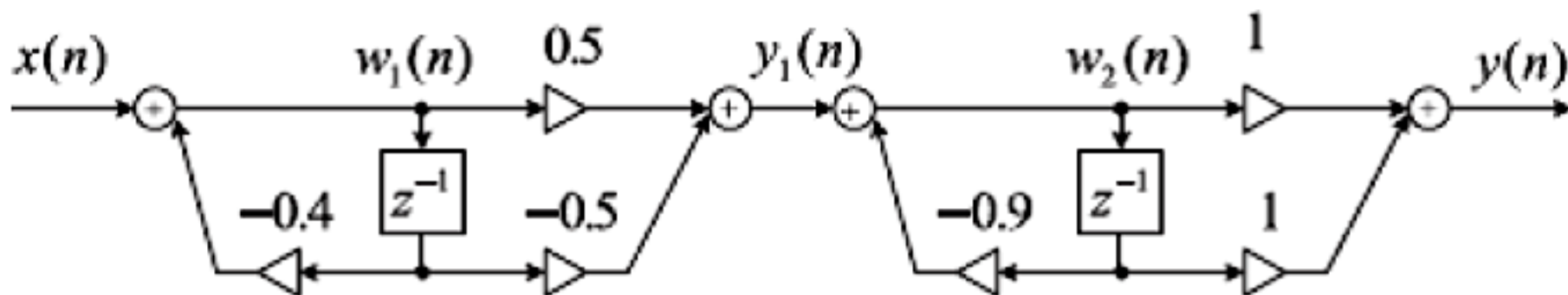
$$H_1(z) = \frac{0.5 - 0.5z^{-1}}{1 + 0.4z^{-1}}$$

$$H_2(z) = \frac{1 + z^{-1}}{1 + 0.9z^{-1}}$$

Cascade (Series) Realization

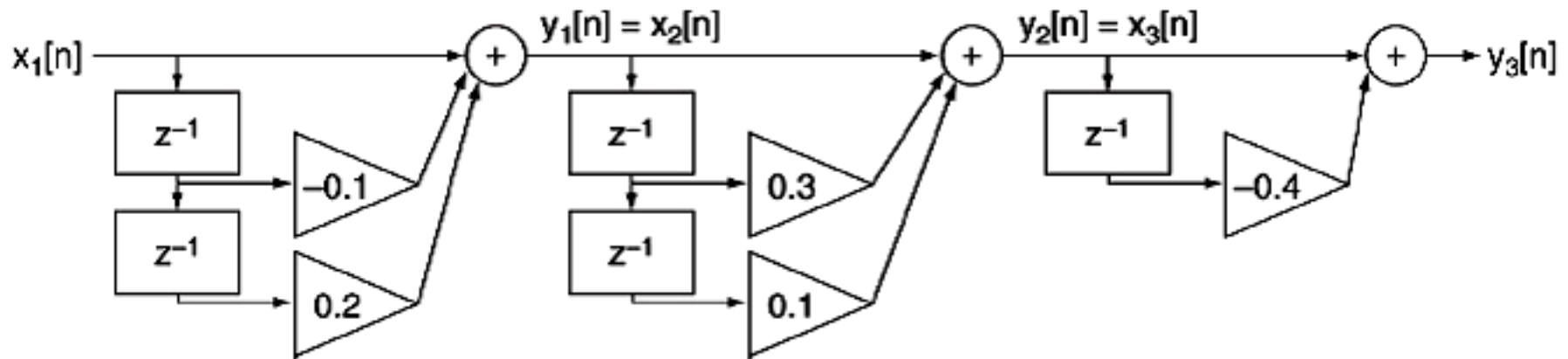
Notice that the obtained $H_1(z)$ and $H_2(z)$ are not the unique selections for realization. For example, there is another way of choosing $H_1(z) = \frac{0.5-0.5z^{-1}}{1+0.9z^{-1}}$ and $H_2(z) = \frac{1+z^{-1}}{1+0.4z^{-1}}$ to yield the same $H(z)$.

Using the $H_1(z)$ and $H_2(z)$ we have obtained, and with the direct-formIII realization, we achieve the cascade form



Cascade (Series) Realization

Example-4: Find the difference equation that corresponds to the cascaded system.



- First write down the difference equation of each stage.

For stage 1: $y_1[n] = x_1[n] - 0.1x_1[n-1] + 0.2x_1[n-2]$

For stage 2: $y_2[n] = x_2[n] + 0.3x_2[n-1] + 0.1x_2[n-2]$

For stage 3: $y_3[n] = x_3[n] - 0.4x_3[n-1]$

Cascade (Series) Realization

- First write down the difference equation of each stage.

For stage 1: $y_1[n] = x_1[n] - 0.1x_1[n-1] + 0.2x_1[n-2]$

For stage 2: $y_2[n] = x_2[n] + 0.3x_2[n-1] + 0.1x_2[n-2]$

For stage 3: $y_3[n] = x_3[n] - 0.4x_3[n-1]$

- Then find the transfer function of each stage.

For stage 1: $H_1(z) = 1 - 0.1z^{-1} + 0.2z^{-2}$

For stage 2: $H_2(z) = 1 + 0.3z^{-1} + 0.1z^{-2}$

For stage 3: $H_3(z) = 1 - 0.4z^{-1}$

Cascade (Series) Realization

- The transfer functions of the three stages are.

For stage 1: $H_1(z) = 1 - 0.1z^{-1} + 0.2z^{-2}$

For stage 2: $H_2(z) = 1 + 0.3z^{-1} + 0.1z^{-2}$

For stage 3: $H_3(z) = 1 - 0.4z^{-1}$

- The overall transfer function of the system is

$$H(z) = H_1(z)H_2(z)H_3(z)$$

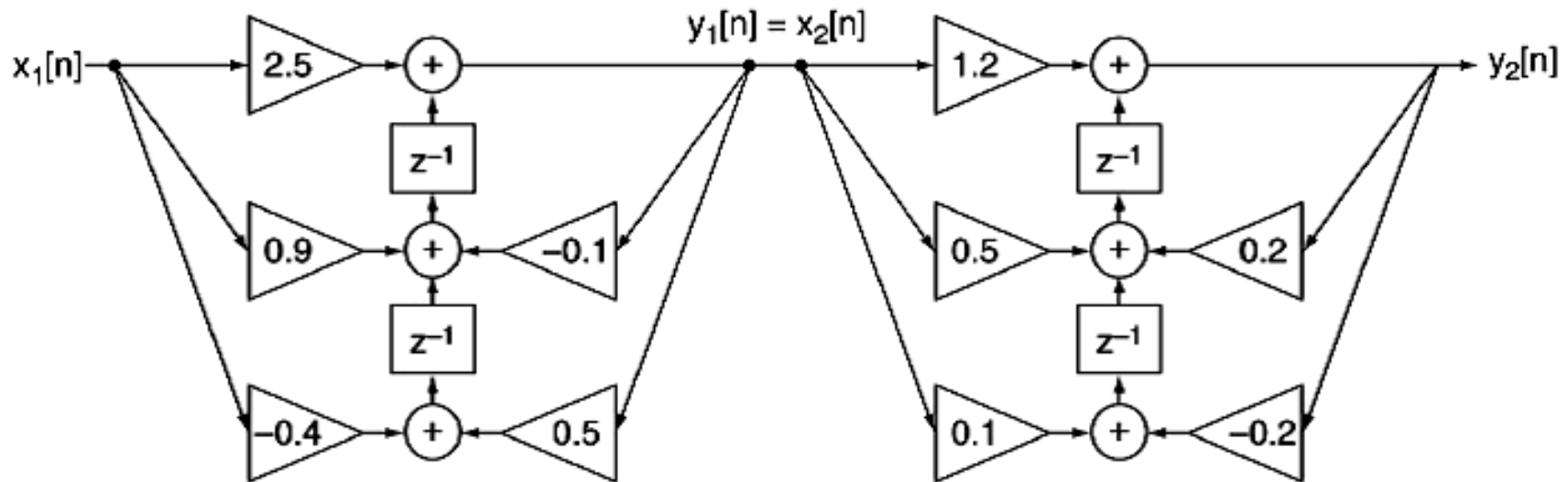
$$H(z) = 1 - 0.2z^{-1} + 0.19z^{-2} - 0.058z^{-3} - 0.008z^{-5}$$

- Finally the difference equation can be get from the $H(z)$

$$Y_3[n] = x_1[n] - 0.2x_1[n-1] + 0.19x_1[n-2] - 0.058x_1[n-3] - 0.008x_1[n-5]$$

Cascade (Series) Realization

Example-5: Find the transfer function of the transposed direct form II realization of the filter.



First write down the difference equation of each stage.

For stage 1:

$$y_1[n] = -0.1y_1[n-1] + 0.5y_1[n-2] + 2.5x_1[n] + 0.9x_1[n-1] - 0.4x_1[n-2]$$

For stage 2:

$$y_2[n] = 0.2y_2[n-1] - 0.2y_2[n-2] + 1.2x_2[n] + 0.5x_2[n-1] + 0.1x_2[n-2]$$

Cascade (Series) Realization

- The difference equation for the filter is most exactly found by multiplying the transfer function for each stage.

$$H(z) = H_1(z)H_2(z)$$

$$H(z) = \left(\frac{2.5 + 0.9z^{-1} - 0.4z^{-2}}{1 + 0.1z^{-1} - 0.5z^{-2}} \right) \left(\frac{1.2 + 0.5z^{-1} + 0.1z^{-2}}{1 - 0.2z^{-1} + 0.2z^{-2}} \right)$$

$$H(z) = \frac{3 + 2.33z^{-1} + 0.22z^{-2} - 0.11z^{-3} - 0.04z^{-4}}{1 - 0.1z^{-1} + 0.32z^{-2} + 0.12z^{-3} - 0.1z^{-4}}$$

Parallel Realization

- Now we convert $H(z)$ into the form:

$$H(z) = H_1(z) + H_2(z) + \cdots + H_k(z) \quad (16)$$

where $H_k(z)$ is defined as the first- or second-order transfer function (section) given by

$$H_k(z) = \frac{b_{k0}}{1 + a_{k1}z^{-1}} \quad (17)$$

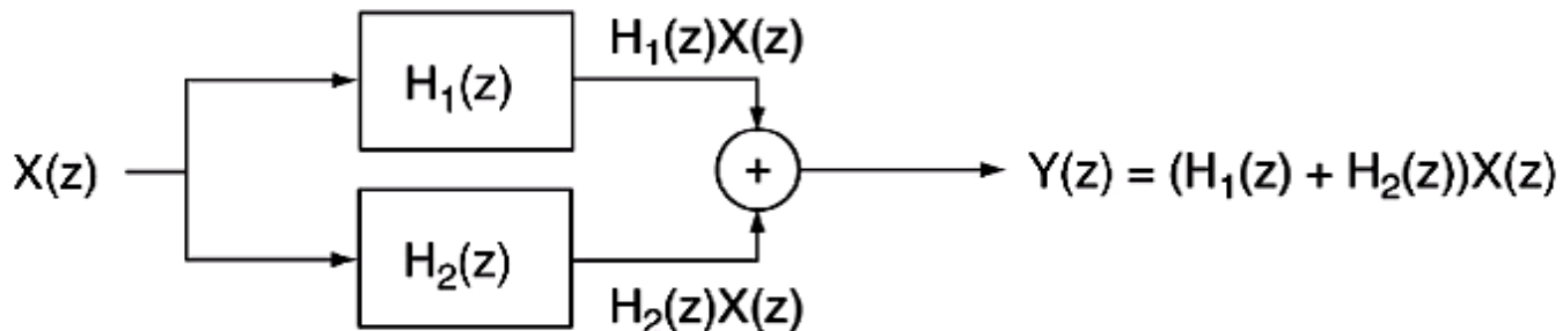
or

$$H_k(z) = \frac{b_{k0} + b_{k1}z^{-1}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}} \quad (18)$$

respectively.

Parallel Realization

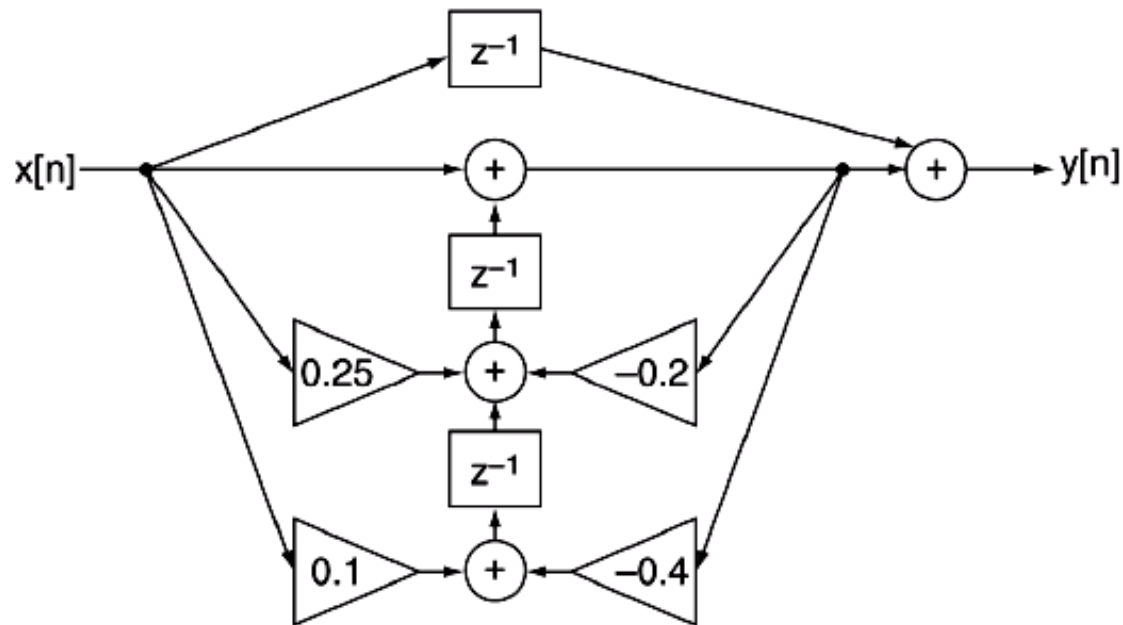
- When two or more systems are combined in parallel the transfer functions of the systems can be used to determine a transfer function for the overall system.
- E.g. The transfer function of the two systems connected in parallel is $H(z) = H_1(z) + H_2(z)$
- The resulting parallel realization is illustrated in the block diagram in following Figure.



(b) Parallel Combination of Filters

Parallel Realization

Example-6: Find the transfer function of the parallel combination.



- **For Top Section:** $H_1(z) = z^{-1}$
- **For Bottom Section:** $H_2(z) = \frac{1+0.25z^{-1}+0.1z^{-2}}{1+0.2z^{-1}+0.4z^{-2}}$

Parallel Realization

- The overall transfer function of the parallel system is

$$H(z) = \frac{1 + 0.25z^{-1} + 0.1z^{-2}}{1 + 0.2z^{-1} + 0.4z^{-2}} + z^{-1}$$

Parallel Realization

Example-7: Draw parallel realization of a second-order transfer function given by

$$H(z) = \frac{0.5(1 - z^{-2})}{1 + 1.3z^{-1} + 0.36z^{-2}}$$

Solution

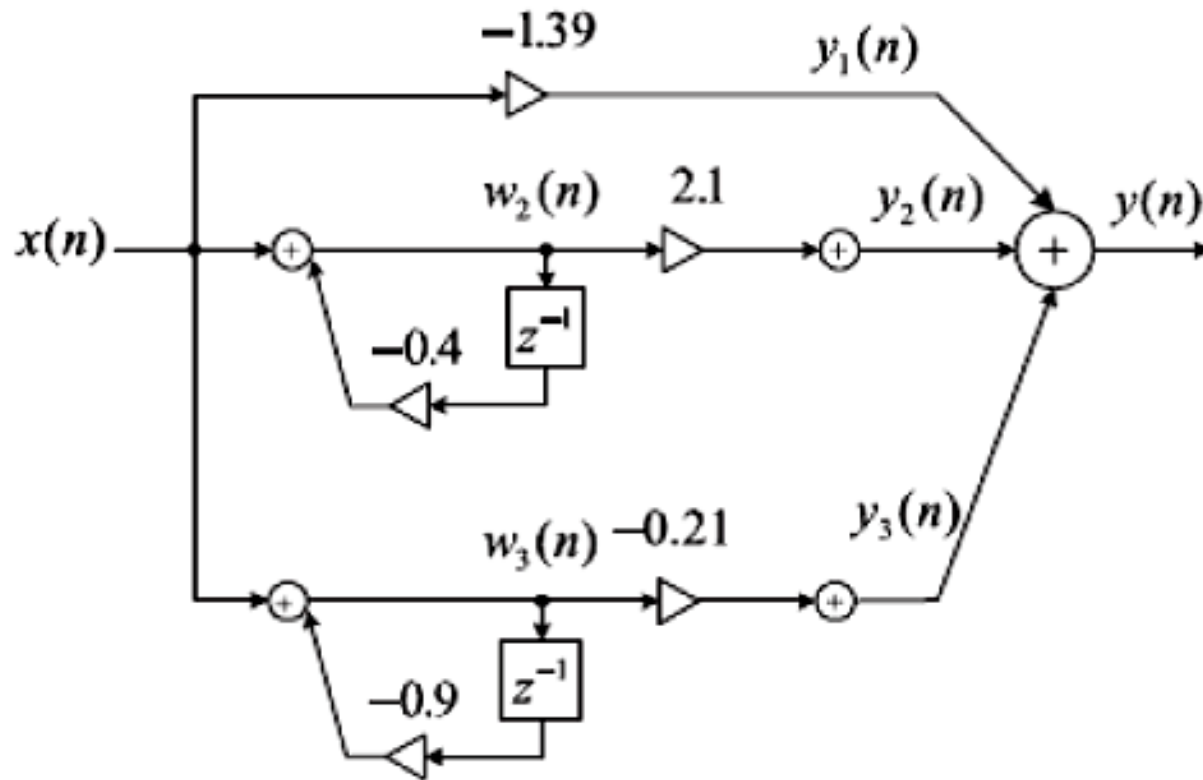
In order to yield the parallel form of realization, we need to make use of the partial fraction expansion. We get

$$H(z) = -1.39 + \frac{2.1}{1 + 0.4z^{-1}} + \frac{-0.21}{1 + 0.9z^{-1}}$$

Parallel Realization

$$H(z) = -1.39 + \frac{2.1}{1 + 0.4z^{-1}} + \frac{-0.21}{1 + 0.9z^{-1}}$$

- using the direct-form II realization for each section, we obtain the parallel realization in Following figure.

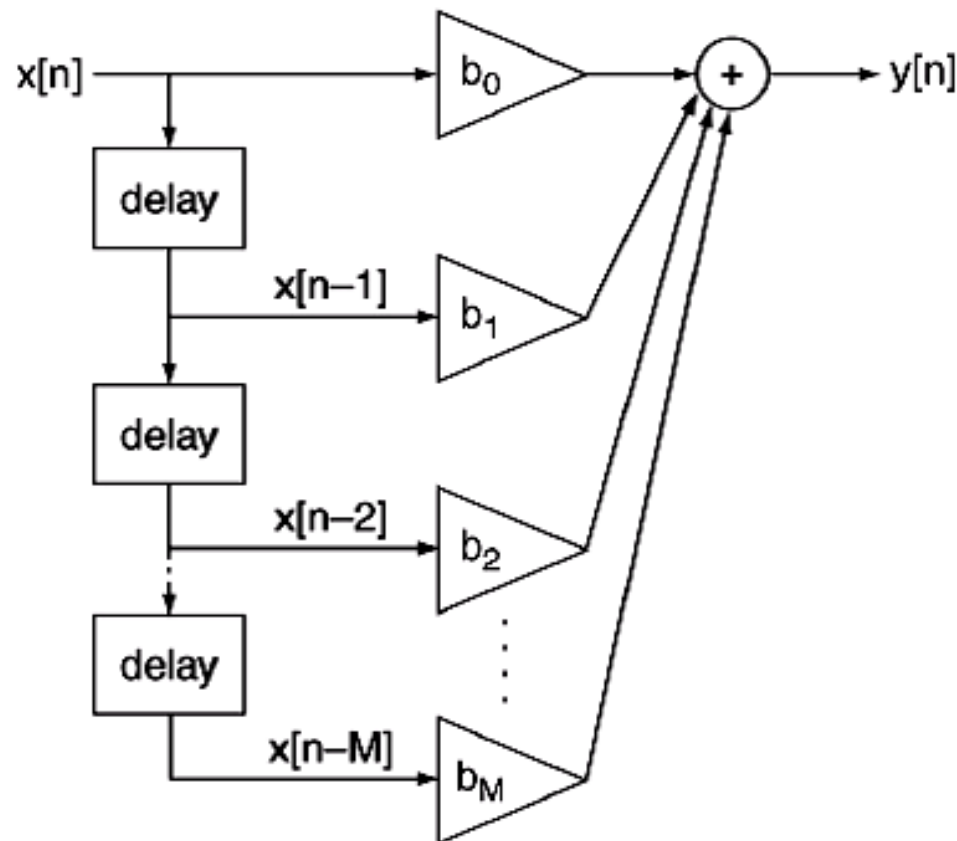


Realization of Digital Systems

- In practice, the second-order filter module with the direct-form I or direct-form II realization is used.
- The high-order filter can be factored in the cascade form with the first- or second-order sections.
- In cases where the first order-filter is required, we can still modify the second-order filter module by setting the corresponding filter coefficients to be zero.

Nonrecursive Difference Equation Diagram

- The general form of the nonrecursive difference equation is
$$y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2] + \dots + b_Mx[n-M]$$
- It can be schematically presented as



Nonrecursive Difference Equation Diagram

- In order to reduce quantization error of filter's coefficient one strategy is to break higher order filter into second order chunks

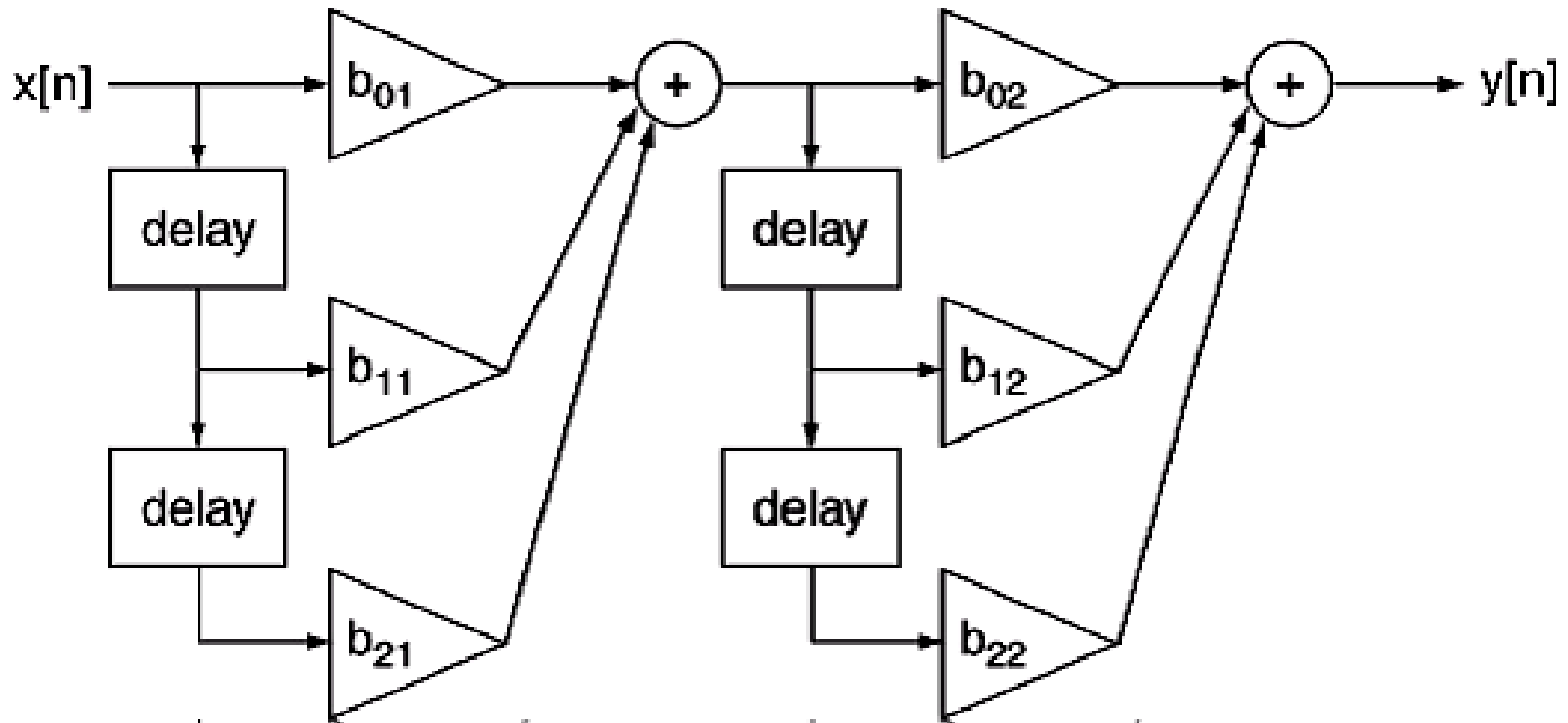


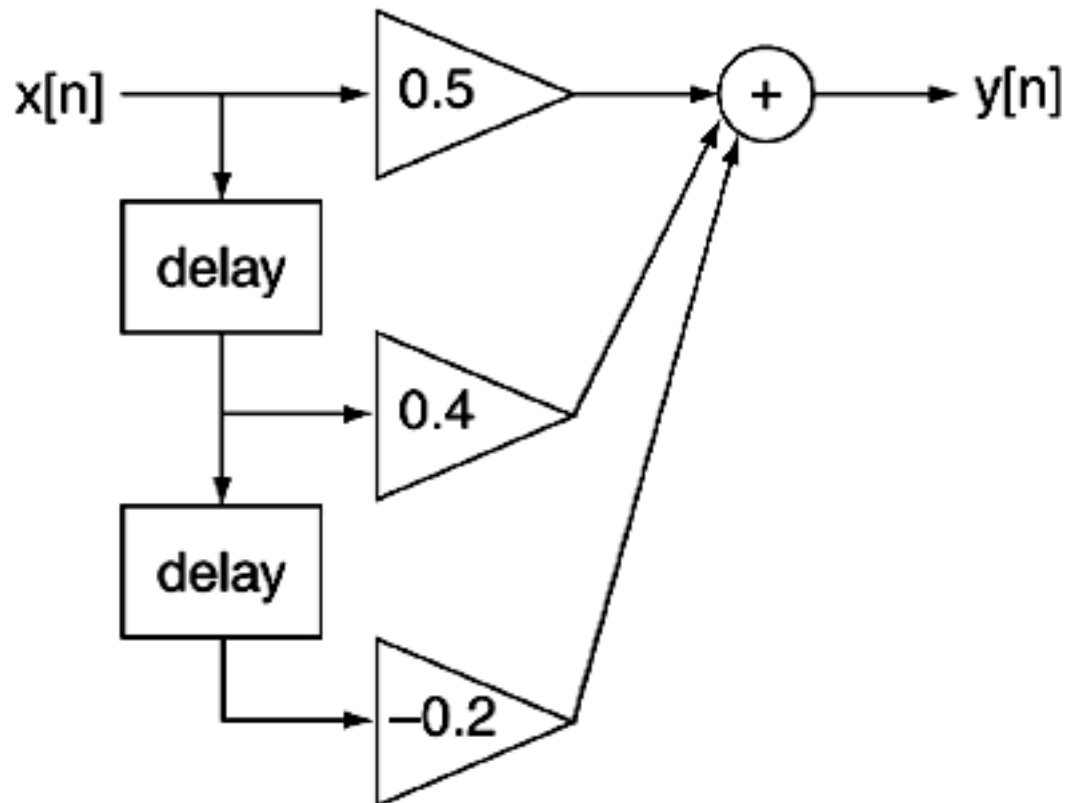
Figure: Cascaded second order nonrecursive filter sections

Nonrecursive Difference Equation Diagram

Example-3: Draw a diagram for the difference equation

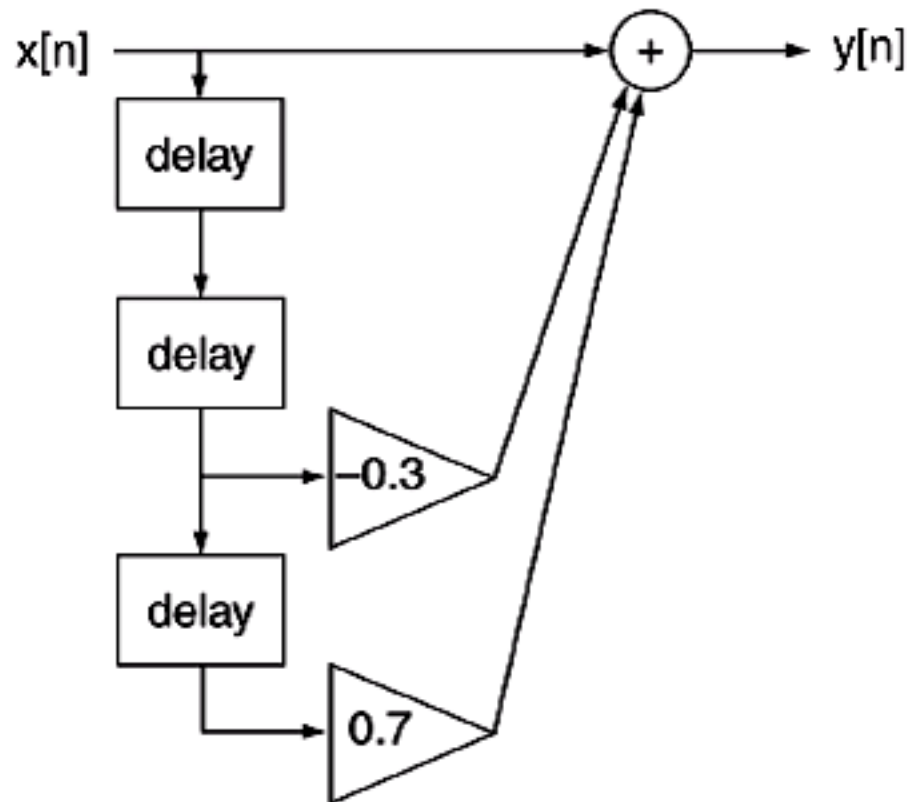
$$y[n] = 0.5x[n] + 0.4x[n-1] - 0.2x[n-2]$$

Solution



Nonrecursive Difference Equation Diagram

Example-4: Write the difference equation of the following diagram.

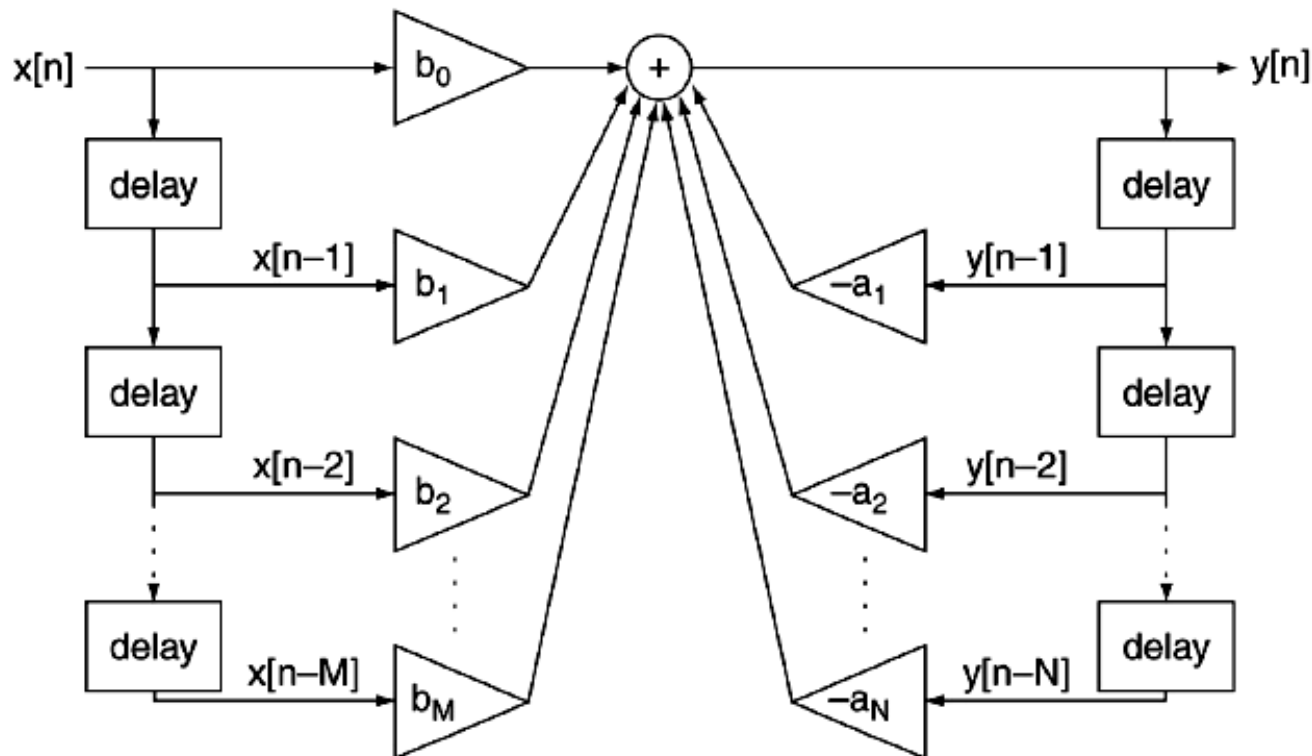


Solution

$$y[n] = x[n] - 0.3x[n-2] + 0.7x[n-3]$$

Recursive Difference Equation Diagram

- The general form of a recursive **difference equation** is
$$a_0y[n] + a_1y[n-1] + a_2y[n-2] + \dots + a_Ny[n-N]$$
$$= b_0x[n] + b_1x[n-1] + b_2x[n-2] + \dots + b_Mx[n-M]$$



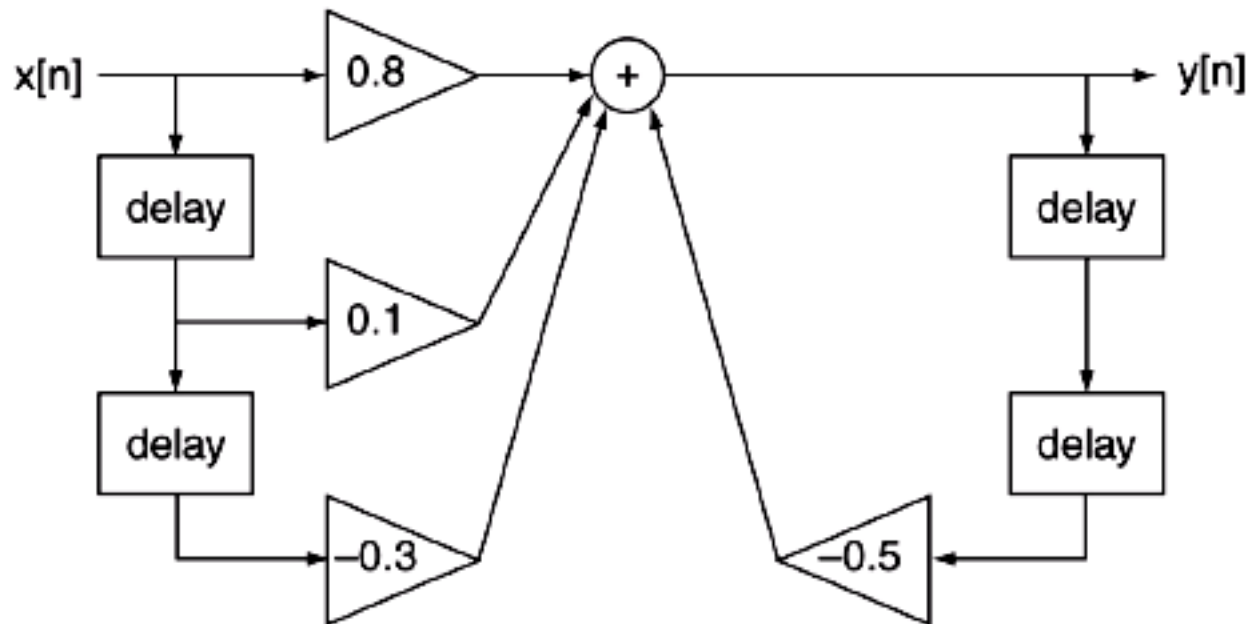
Recursive difference equation diagram, direct form 1 realization.

Recursive Difference Equation Diagram

Example-5: Draw a direct form I diagram for the following recursive difference equation

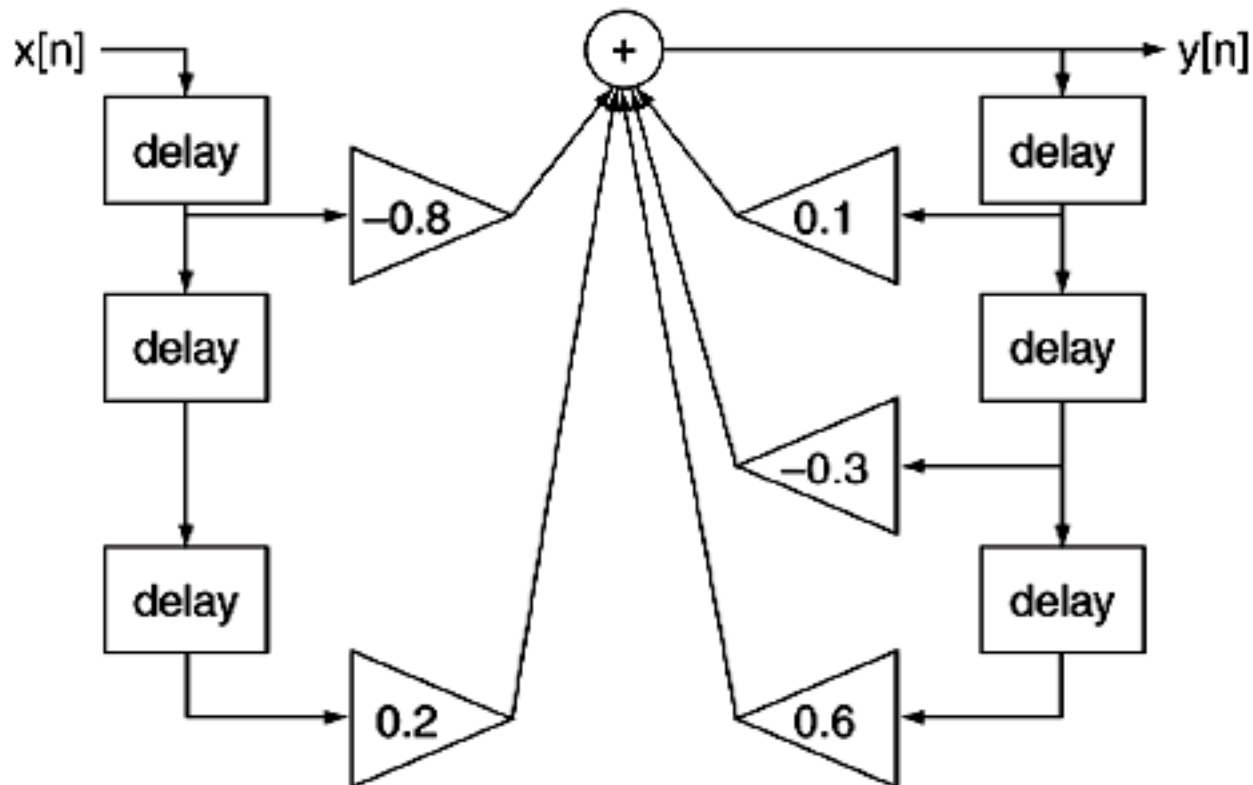
$$y[n] + 0.5y[n-2] = 0.8x[n] + 0.1x[n-1] - 0.3x[n-2]$$

Solution



Recursive Difference Equation Diagram

Example-6: Write the difference equation of the following diagram.



Solution

$$y[n] + 0.5y[n-2] = 0.8x[n] + 0.1x[n-1] - 0.3x[n-2]$$

Recursive Difference Equation Diagram

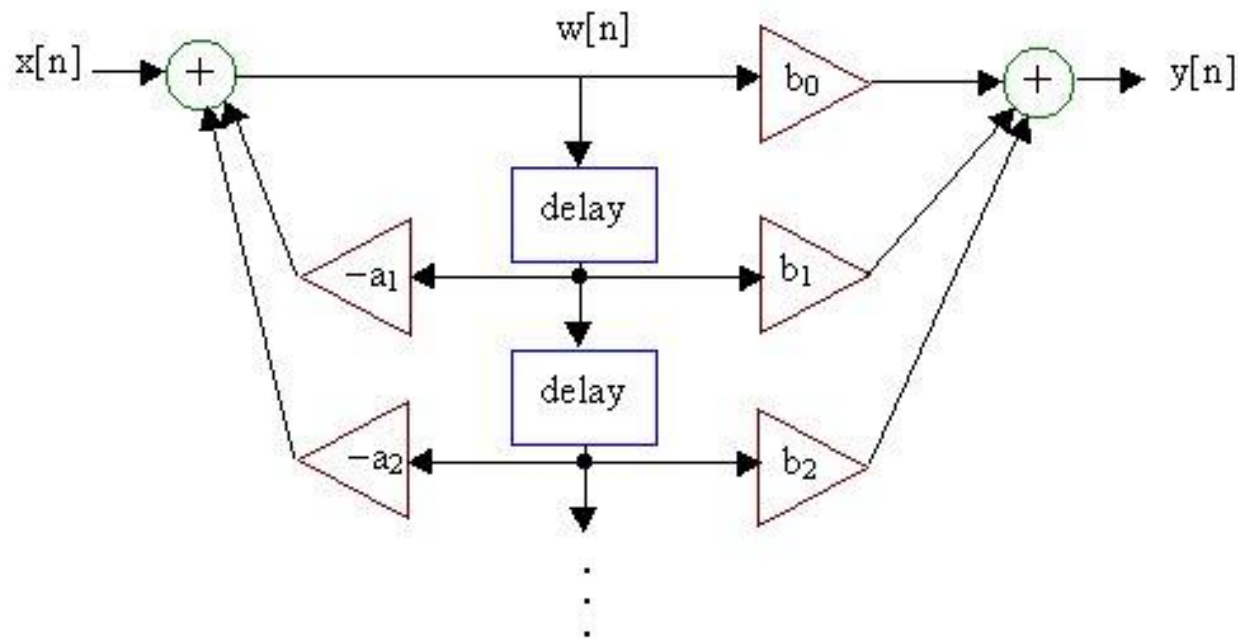
- In **direct form I realization**, calculation for output $y[n]$ require
 - $M + 1$ input states
 - N output states
 - $M + N + 1$ coefficient multiplications
 - $M + N$ additions
- When more than 2 or 3 delays are needed, this realization is very sensitive to the finite word length effects.

Recursive Difference Equation Diagram

- Recursive filters have an alternative representation, called **direct form 2**, that has certain advantages for implementation.
- This representation is defined by the pair of equations

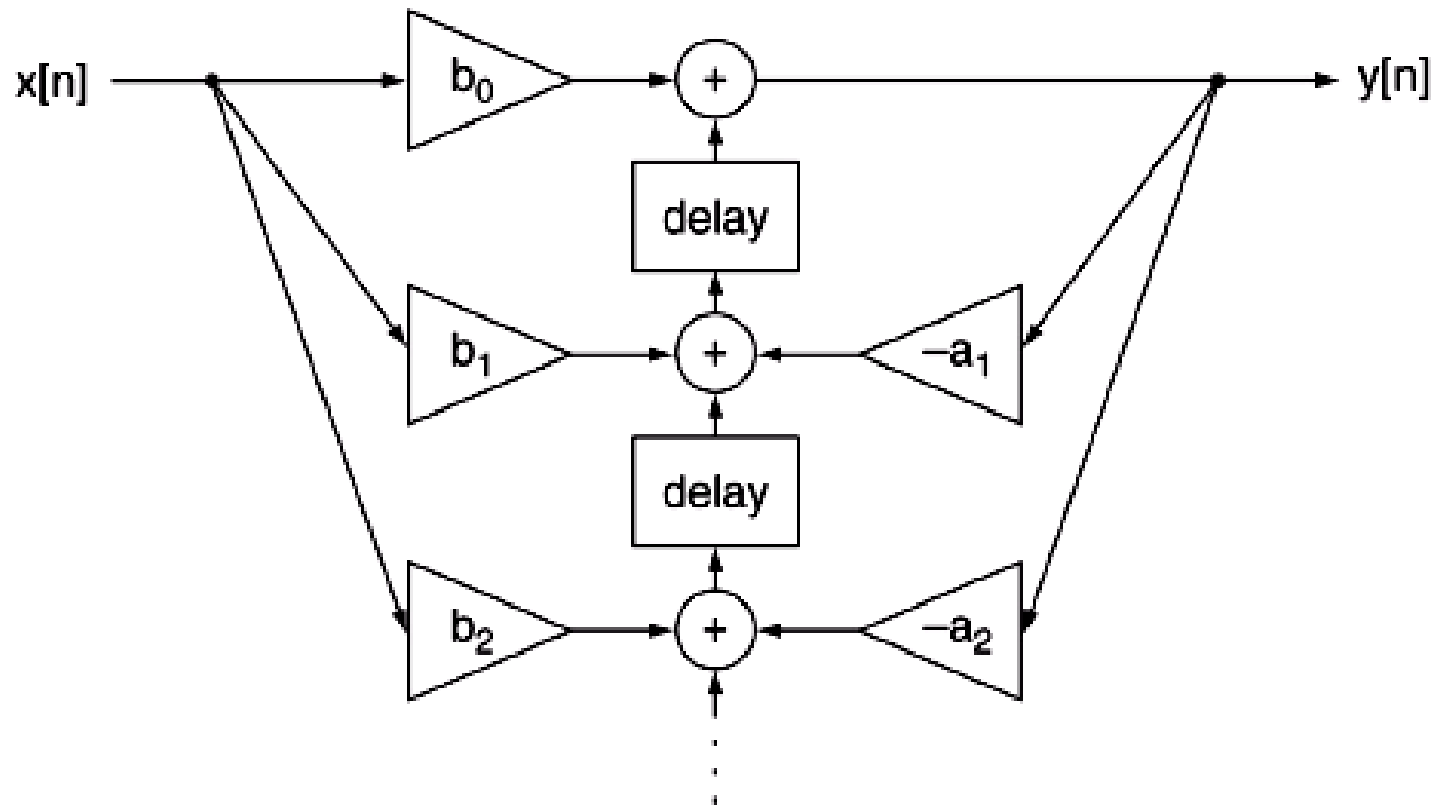
$$w[n] = x[n] - a_1 w[n-1] - a_2 w[n-2] - \dots - a_N w[n-N]$$

$$y[n] = b_0 w[n] + b_1 w[n-1] + b_2 w[n-2] + \dots + b_M w[n-M]$$



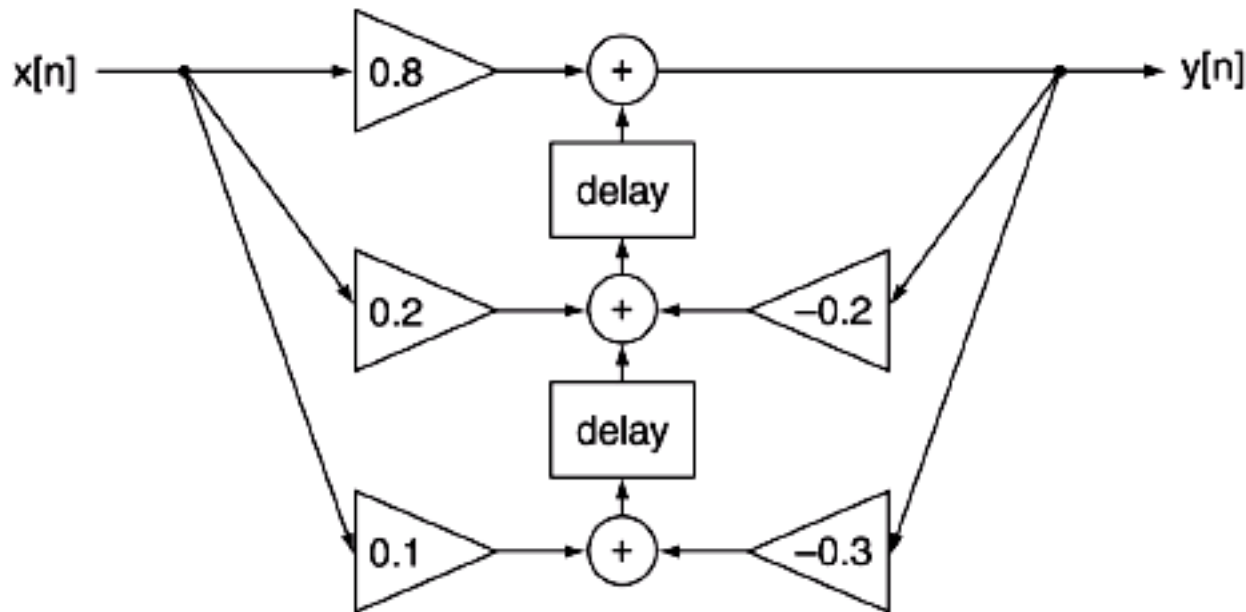
Recursive Difference Equation Diagram

- Transpose of the direct form 2 realization is an another popular implementation model.



Recursive Difference Equation Diagram

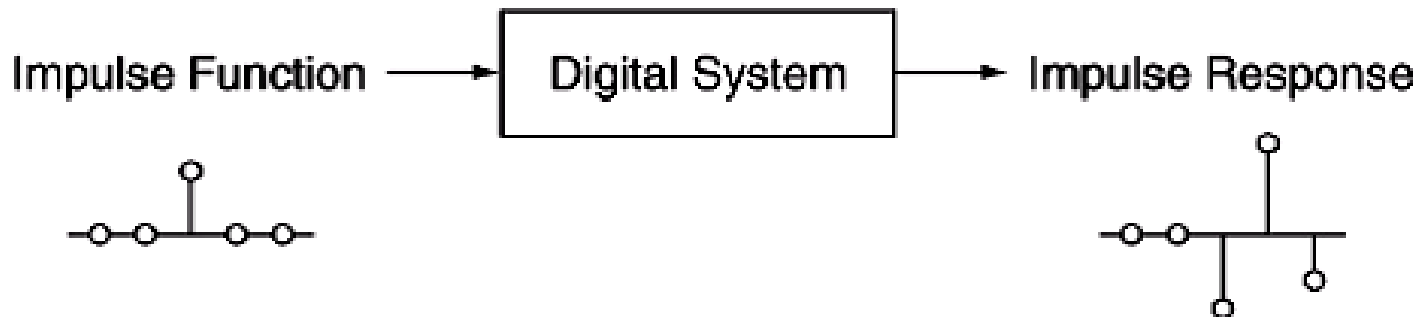
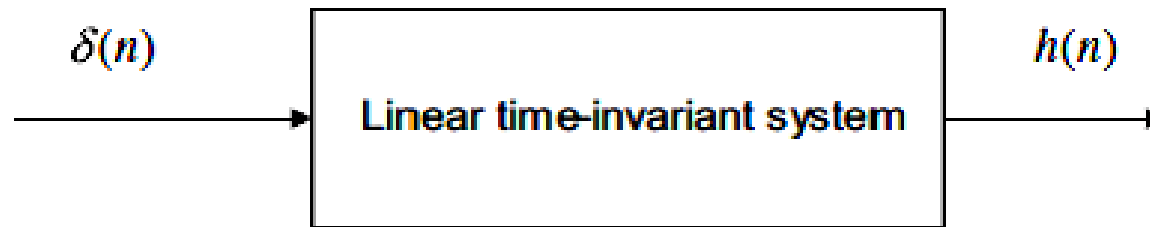
Example-7: Write the difference equation of the following diagram.



- The output of the bottom summer is $0.1x[n] - 0.3y[n]$
- The output of the middle summer is $0.1x[n-1] - 0.3y[n-1] + 0.2x[n] - 0.2y[n]$
- The final out top summer is $y[n] = 0.1x[n-2] - 0.3y[n-2] + 0.2x[n-1] - 0.2y[n-1] + 0.8x[n]$

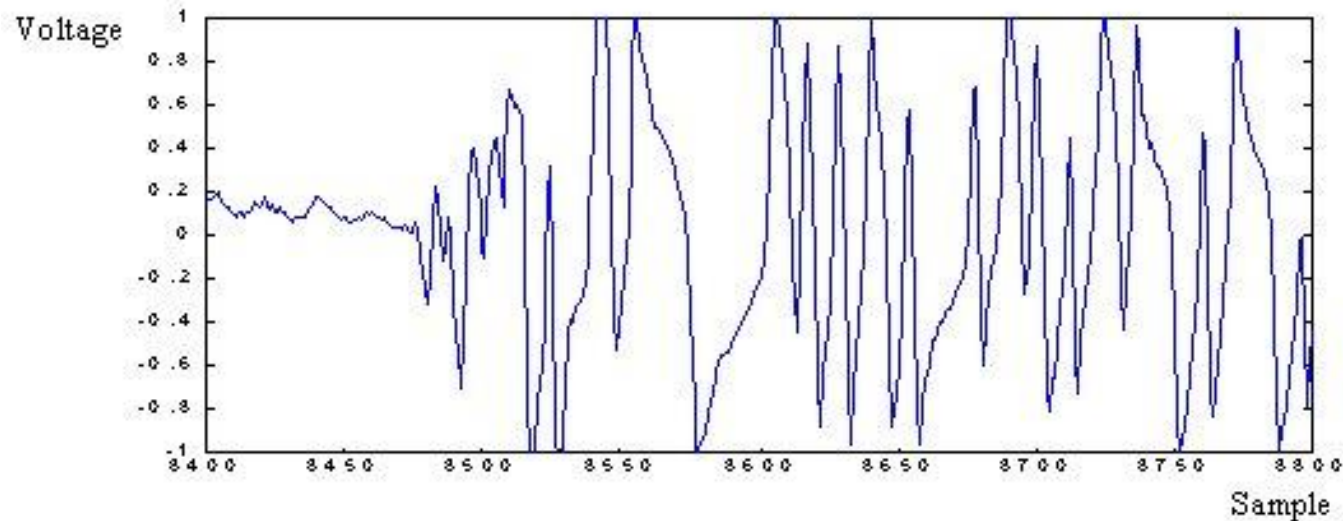
Impulse Response

- The impulse response $h[n]$ is an important way of characterizing a filter or system.
- By definition, it is the system's response to an impulse function input $\delta[n]$.



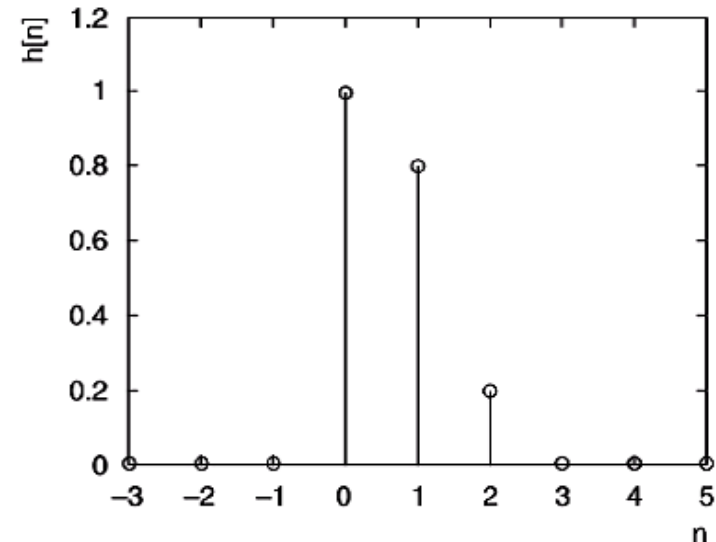
Impulse Response

- For a musical instrument like a piano, an impulse response corresponds to the note obtained by striking a single key.



Impulse Response

Example-9: write the difference equation whose impulse response is shown in the figure.



Solution

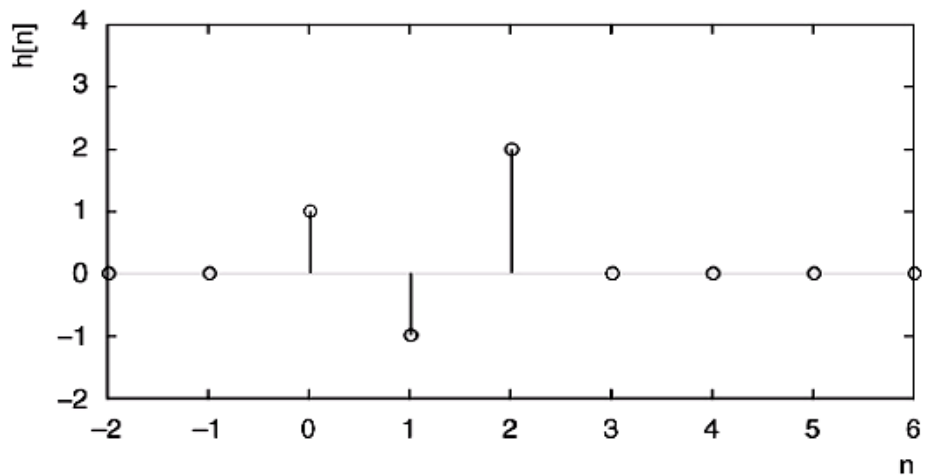
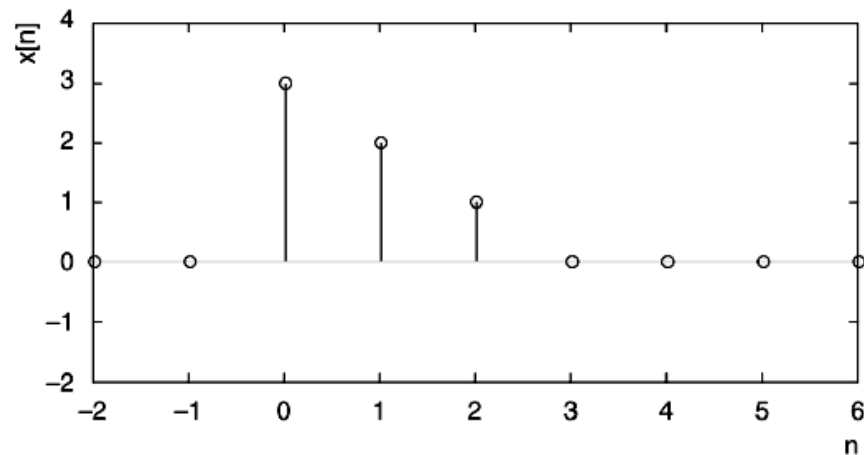
The impulse response can be written as a sum of impulse functions.

$$h[n] = \delta[n] + 0.8\delta[n-1] + 0.2\delta[n-2]$$

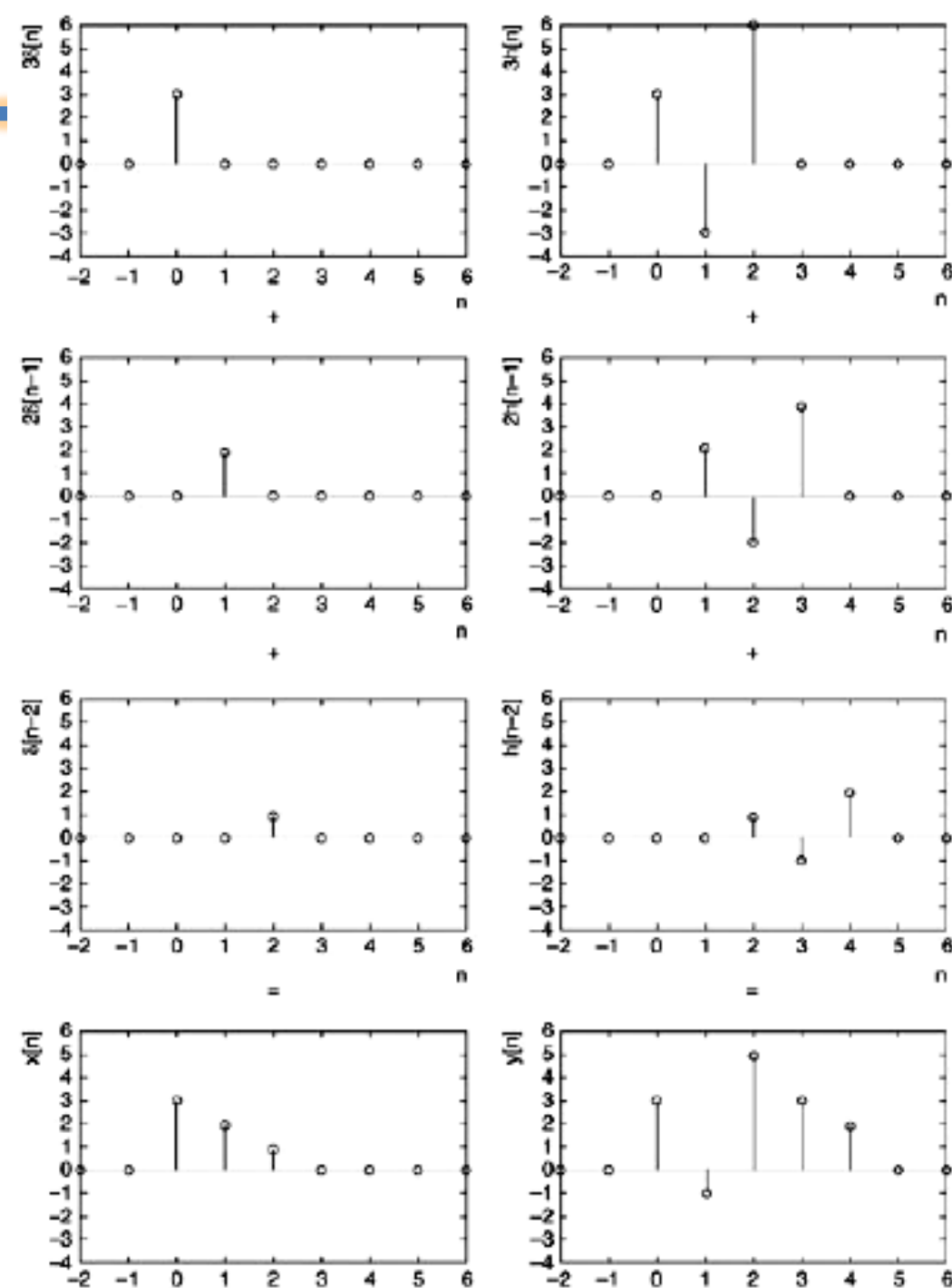
The difference equation is $y[n] = x[n] + 0.8x[n-1] + 0.2x[n-2]$

Impulse Response

Example-10: The input signal $x[n]$ and impulse response $h[n]$ are shown in the figures. Find the output $y[n]$ by breaking the input signal into impulse functions and finding the response to each.



Impulse Response



$$Y[n] = 3h[n] + 2h[n-1] + h[n-2]$$

Step Response

- The step response $s[n]$ for a filter is its response to a step input $u[n]$.
- It records the response of the system to a change in level, and may be calculated in two ways.
- The first way mimics that for the impulse response, but with $s[n]$ replacing $y[n]$ and $u[n]$ replacing $x[n]$.

Step Response

- For non-recursive systems,

$$s[n] = b_0u[n] + b_1u[n-1] + b_2u[n-2] + \dots + b_Mu[n-M]$$

- For recursive systems,

$$s[n] = -a_1s[n-1] - a_2s[n-2] - \dots - a_Ns[n-N] \\ + b_0u[n] + b_1u[n-1] + b_2u[n-2] + \dots + b_Mu[n-M]$$

Step Response

- The step function $u[n]$ is equivalent to a sum of impulse functions:

$$u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \dots$$

- The step response $s[n]$ is the same sum of impulse responses:

$$s[n] = h[n] + h[n-1] + h[n-2] + \dots$$

Step Response

- The second way to compute the step response $s[n]$ relies on the impulse response.
- each step response sample is a cumulative sum of the impulse response samples.
- **For example**

$$s[4] = h[4] + h[3] + h[2] + h[1] + h[0]$$

Step Response

Example-11: Find the first eight samples of the step response $s[n]$ for the difference equation

$$Y[n] - 0.2y[n-1] = 0.5x[n] + 0.3x[n-1]$$

Solution

The step response $s[n]$ of the difference equation is

$$s[n] - 0.4s[n-1] = u[n] - u[n-1]$$

So the first eight samples of the step response are

$s[0] = 0.5$	$s[3] = 0.96$	$s[6] = 0.996$
$s[1] = 0.8$	$s[4] = 0.98$	$s[7] = 0.9984$
$s[2] = 0.9$	$s[5] = 0.992$	

Step Response

So the first eight samples of the step response are

$s[0] = 0.5$	$s[3] = 0.96$	$s[6] = 0.996$
$s[1] = 0.8$	$s[4] = 0.98$	$s[7] = 0.9984$
$s[2] = 0.9$	$s[5] = 0.992$	

$$S[n] = [0.5, 0.8, 0.9, 0.96, 0.98, 0.992, 0.996, 0.9984]$$

